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A Computationally Efficient Formulation to Accurately Represent Start-Up Costs in the Medium-Term Unit Commitment Problem

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***Abstract*—Nowadays, the changing paradigm in power systems highlights the necessity of improving detail in energy models. The deployment of non-dispatchable renewable energy resources that regulators traditionally promoted, like the European Union case and some areas of the United States, is being accelerated due to the enhanced competitiveness of clean technologies. However, the delay in the widespread use of non-conventional energy-storage techniques, like batteries, is causing an increase in the variability of the thermal-generated demand for electricity. Moreover, many coal-fired plants are being dismantled as a result of their end-of-life, new climate change policies, and high prices in the emissions allowances trading markets. Meanwhile, nuclear energy remains a backup generator but does not offer the possibility of boosting its operational flexibility. Thereby, combined cycles gas turbines are positioned as a key vector towards a clean energy transition, increasing their start-up and shut-down frequency for operating a lower number of hours than historically have done. Their fast ramping capabilities position them as the best alternative to cover demand peaks in the near term. Consequently, there is a growing interest in representing these thermal units properly. This paper exposes a great-detailed analysis of the start-up cost modeling to accomplish future market trends. Likewise, a new mathematical formulation of the unit commitment problem is also presented. Finally, the formulation is compared to one of the most renowned methodologies. Its successful performance is described in several case studies, where authentic power-demand curves and technical details of a gas-fired generation portfolio are handled in medium-term horizons.**

***Index Terms*—unit commitment, start-up cost representation, piecewise linearization, stairwise aggregation method, medium-term representation, power systems, electricity markets, efficient formulation, optimization models.**

Nomenclature

A. Sets

$g \in G$	Set of indexes of generating units.
$s \in S$	Set of indexes of start-up segments.
$t \in T$	Set of indexes of hourly periods of the time span.

B. Parameters

C_g^F	Fuel cost of unit g [\$/MMBtu].
C_g^{P-M}	Linear variable production cost of unit g [\$/MWh].
C_g^{P-N}	Fixed production cost of unit g [\$/h].
C_g^{SD}	Shut-down cost of unit g [\$].
$C_{g,s}^{SU-M}$	Linear variable cost for the start-up type s of unit g [\$/h].
$C_{g,s}^{SU-N}$	Fixed cost for the start-up type s of unit g [\$].
D_t	Load demand in period t [MWh].
F_g^{P-M}	Linear variable fuel-consumption for producing electricity of unit g [MMBtu/MWh].
F_g^{P-N}	Fixed fuel-consumption for producing electricity of unit g [MMBtu/h].
F_g^{SD}	Shut-down fuel-consumption of unit g [MMBtu].
$F_{g,s}^{SU-M}$	Linear variable fuel-consumption for the start-up type s of unit g [MMBtu/h].
$F_{g,s}^{SU-N}$	Fixed fuel-consumption for the start-up type s of unit g [MMBtu].
$\overline{H}$	High value to discern in the start-up type decisions.
$\overline{P_g}$	Maximum power output of unit g [MW].
$\underline{P_g}$	Minimum power output of unit g [MW].
$\overline{R_t}$	Spinning reserve requirement in period t [MWh].
RD_g	Ramp-down limit of unit g [MW/h].
RU_g	Ramp-up limit of unit g [MW/h].
SD_g	Shut-down capability of unit g [MW].
SU_g	Start-up capability of unit g [MW].
$T_{g,s}^{SU}$	Minimum time period that unit g must be offline for the start-up type s [h].
TD_g	Minimum down time of unit g [h].
TU_g	Minimum up time of unit g [h].

C. Variables

1) Positive and continuous variables

$c_{g,t}^P$	Production cost of unit g in period t [\$].
$c_{g,t}^{SD}$	Shut-down cost of unit g in period t [\$].
$c_{g,t}^{SU}$	Start-up cost of unit g in period t [\$].
$h_{g,t}^{SD}$	Number of hours that unit g has been offline in period t [h].
$h_{g,t,s}^{SU}$	Number of hours that unit g has been offline individualized to the segment s in which unit g starts-up in period t [h].
$p_{g,t}$	Power output above the minimum output of unit g in period t [MW].
$r_{g,t}$	Spinning reserve served by unit g in period t [MW].

2) Binary variables

$u_{g,t}$	Commitment decision of unit g in period t.
$v_{g,t,s}$	Start-up decision of unit g in period t and type s.
$w_{g,t}$	Shut-down decision of unit g in period t.

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I. Introduction

THE operation trends of many power systems are currently changing due to the high penetration of non-dispatchable renewable energy resources. Consequently, demand variability of electricity produced by dispatchable generators has notably increased in recent years. Additionally, market analyses agree on an upcoming variability increment until the widespread use of energy-storage technologies [1]–[3].

These facts bring attention to fast-response technologies to guarantee the security of supply. On one side, hydro generation can solve sudden demand variations. On the other hand, water availability is not unlimited, and its capacity can not often be boosted in most mature power systems, where it is frequently entirely leveraged. Therefore, fast-ramping thermal generators are positioned as the best alternative to satisfy demand peaks [4], as the importance of improving the operational flexibility is gaining ground [5].

Optimizing the management of generation portfolios entails an essential task for market players and system operators. For that reason, there is a growing interest in increasing the detail of energy models [6]. The unit commitment problem facilitates an accurate representation of technical and economic aspects of power systems. However, dealing with great detail involves a high computational demand [7].

Accordingly, efficient formulations have continuously been proposed in the literature [8]–[22] to enhance the performance of commercial solvers. These tools habitually manifest a more proficient response to certain mathematical representations of the same modeled concern. Regarding the unit commitment, a great variety of modeling options are available to address the problem. Nevertheless, the utilization of Mixed Integer Linear Programming (MILP) formulations provides one of the most computationally-efficient resolution processes [22], [23].

Furthermore, MILP formulations guarantee the convergence to the optimal solution in a finite number of steps [24]. This mathematical representation requires the usage of linearization techniques to model those features whose behavior is defined through non-linear functions. Moreover, although the current commercial solvers can successfully manage convex and non-convex quadratic programs, computational execution is usually better when these functions are linearized [25].

Nonetheless, the methodological adequacy for handling the unit commitment problem can not be determined categorically. Even though the application of some formulating principles is beforehand favorable, computational efficiency is not always predictable. It requests constant research because it is related to the state-of-art of resolution techniques. A clear example is [11], where some binary variables, which are difficult to solve, are replaced by extra linear constraints in order to accelerate the resolution process. After that, [13] employed a new version of the solver and demonstrated that it exploits binary variables for enhancing the branching process and generating better cuts, improving the work on the enumeration tree and the run times.

In that way, reducing variables and constraints at most does not always mean a resolution upgrading, nor the addition of a vast amount of extra inequalities to tighten the problem. (Tight formulations try to approximate the relaxed feasible region to the integer one to speed the convergence towards the optimal solution). The inclusion of additional variables and constraints to eliminate integer-infeasible regions in the relaxed polytope frequently helps the solver [26]. Nevertheless, [15] highlights that a trade-off between tightness and compactness (avoiding big-size relaxed problems that demand higher resolution times and hinder the enumeration tree exploration) is mandatory.

The tight and compact formulation proposed in [16] for the energy-based unit commitment problem offers one of the best computational performances nowadays. This methodology has been recently compared to other renowned formulations, like [19]–[21], and has demonstrated its validity [21]–[23].

Naturally, some formulations respond better to certain case studies and commercial solvers. On the one hand, literature is plenty of case studies to test the effectiveness of the proposed methodologies. On the other, some authors resort to the same instances to provide a clear comparison benchmark. However, it is very common to appreciate that either the technical detail, demand curves, or time spans are over-simplified accordingly to the size and complexity of real power systems.

Regarding this matter, start-up costs frequently constitute a misrepresented feature of the unit commitment problem. As it is analyzed in [27], most of the literature uses just one or two start-up steps to model this operation. This approach could be appropriate when the thermal units were started-up to generate electricity throughout the whole week or working days. Even so, the above-mentioned current market trends are demanding greater modeling detail. Some recent articles present enhanced representations of start-up features, like power trajectories and configuration transitions [28] or dynamic ramping [29]. In any case, there is still a gap in accurately modeling start-up costs.

The non-linear behavior of this cost, which is described by an exponential function of the offline time prior to the start-up, complicates the detail because of its inherent non-convexity. Nonetheless, this limitation can be easily overcome through a stairwise linearization [9]. In turn, technical information about real start-up processes does not abound in the literature.

Some papers that bring parameters of the exponential curves are [30]–[32]. These functions are often flattened to the coldest start-up after being offline for ~10 hours, which can be enough for small power plants. However, real-size combined cycle gas

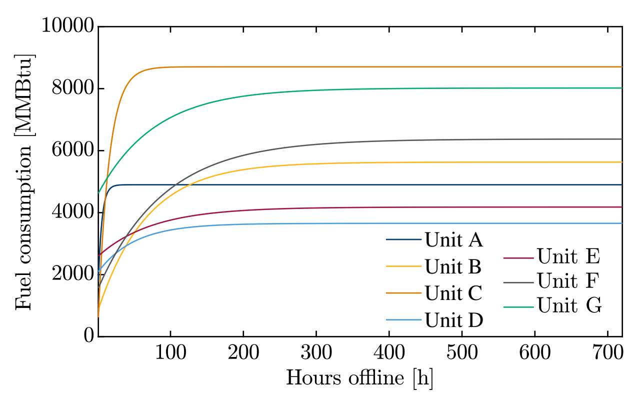

Fig. 1. Real start-up fuel-consumption curves of a thermal portfolio composed of seven combined cycle gas turbine power plants, depending on the number of hours that each unit has been offline.

turbine facilities can last weeks in getting completely cold, as shown in Fig. 1, where the start-up data corresponding to the experts' knowledge about a real thermal portfolio is illustrated.

These *long* start-up curves, together with the increase in the demand variability and the necessity of combining the thermal operation to some other issues, such as fuel purchases or hydro management, request the extension of the horizon in the unit commitment problem, which has been traditionally limited to the short term due to computational requirements.

Medium-term models typically comprise one-month to three years horizons. In those time frames, renewable generation is subject to a high degree of uncertainty, especially at those time steps further from the initial period. That is also the situation of other risk variables, such as electricity demand or unplanned power outages. In that context, efficient problem formulations become essential when probabilistic approaches are required.

Therefore, the ongoing operational trends in power systems with a high penetration of non-dispatchable renewable energy resources need a more precise representation of the actual load profiles in thermal generation. Anyhow, despite the availability of various innovative forecasting techniques, also for the mid-term (e.g. [33]), conservative load profiles are frequently used when novel formulations of the unit commitment problem are tested. Most of these methodologies use 24-hour case studies. Nevertheless, 168-hour instances are also run in [16] and [20].

The repetition of daily-consistent demand patterns has been usually employed in the literature to extend the time span, as

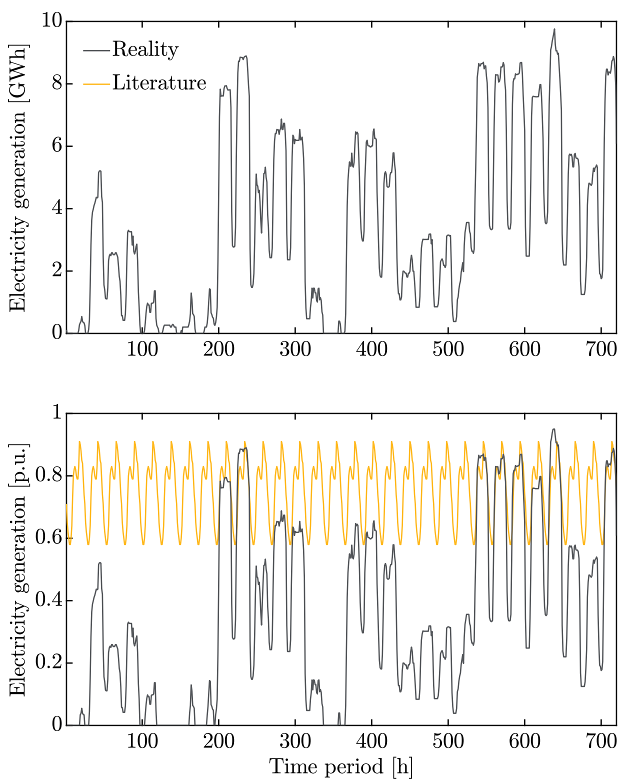

Fig. 2. The gas-fired generation in the Iberian Electricity Market in November 2020 is represented at the top on an hourly basis. Afterwards, this real profile is scaled to a per-unit system at the bottom of the figure, and it is compared to a daily-pattern repetition frequently used in the unit commitment literature. Both series are illustrated in grey and yellow, respectively.

manifested in Fig. 2. Withal, this reiteration does not stress the solver when optimizing the operation in medium- or long-term horizons on an hourly basis. Thus, the resolution performances of the case studies can not be computationally extrapolated to representations of modern electricity markets.

The renowned formulation presented in [16] uses a weekly demand curve similar to the profile illustrated in Fig. 2 except from scaling to an 80% the load pattern at the weekend hours. Meanwhile, the well-known formulation proposed in [20] does not repeat a daily profile. Nonetheless, it shows a conservative curve that does not descend from 50% of the total generation capacity at any time. This behavior can simulate the operation of thermal-prevalent systems where the utilities have not yet backed the widespread deployment of renewable technologies. Nevertheless, this circumstance contrasts the reality of modern power systems, which request the representation of some hours without any thermal generation, exhibiting an increment of the start-up and shut-down processes.

This paper deals with real start-up fuel-consumption curves and proposes a new efficient formulation to work with accurate piecewise linearizations, instead of the conventional stairwise representations, in Section II. Furthermore, this methodology is compared against one of the most renowned formulations in Section III, where a significant run time reduction is exposed and its effectiveness to handle real demand curves in medium-term horizons is demonstrated. Finally, conclusions are shown in Section IV.

The main contributions of this paper are summarized in the following list, explaining the gaps and difficulties appreciated in the literature and the benefits of the proposed methodology:

- Piecewise functions have been usually applied in the unit commitment problem for representing production costs in linear optimization. This approximation imposes linear relationships between power output and cost by bounding generation segments. Nevertheless, the fuel consumption dependence on offline hours leads to non-linear relations between decision variables if the conventional piecewise approach is implemented. To overcome these limitations, a new mathematical formulation is presented to adequate piecewise start-up costs to linear optimization problems.
- Different efficient equations to model start-up costs using flexible step-functions are introduced in several renowned formulations, like [16], [19], [20]. The approach proposed in this paper is compared to the well-known methodology of [16]. This approach takes equations from [16] to model those technical features unrelated to start-up implications. Accordingly, a rigorous analysis of the representation of the start-up details is performed without interference from any other modeling difference. Moreover, the formulation is open to adding or substituting some equations to reach a tighter and more compact problem, like the strengthened ramping constraints declared in [34].
- The ongoing increment of the generation intermittence in electricity markets is captured in the case studies exposed in this article, subjecting the formulation to actual system trends. In addition, the utilization of hourly medium-term time spans (one-month horizons) allows dealing with real start-up curves and accurately studying their performance.

II. METHODOLOGY

The following methodology lets a computationally efficient representation of the start-up processes accounted by thermal generators with high accuracy. An alternative to the traditional stairwise linearization of start-up costs is exposed in this paper, implementing piecewise-linear functions to model this feature.

Piecewise linearizations have been widely employed for the representation of production costs [8]. These linearizations add binary variables into the segments to match power output with the corresponding piecewise block's capacity limits, defined as parameters. Nonetheless, the necessity of using the number of offline hours prior to the start-up, modeled with continuous or integer variables, must not be directly handled through binary segments if linearity is wanted to be preserved.

For that reason, indirect relationships are developed in this formulation to allow the application of piecewise functions to model the start-up costs, guaranteeing that the MILP-behavior is maintained. Besides, this publication focuses on the start-up detail characterization in the medium-term energy-based unit commitment problem. However, this methodology must not be exclusively limited to that concern if an accurate operational modeling of thermal generation is desired.

Accordingly, the formulation presented in Section II-A takes equations from [16] as a benchmark to represent the technical features of the thermal unit commitment problem, and substitutes the start-up constraints by a new proposal. As previously mentioned, the tight & compact methodology apparently offers the best computational performance [23], which is wanted to be taken advantage of in this paper.

Afterwards, Section II-B briefly describes the data curation process to manage real fuel-consumption functions in this formulation and the equivalent procedure to reach the great detail start-up modeling parameters to be subsequently employed in [16] when both representations are compared.

A. Mathematical Formulation of the Unit Commitment

The following formulation presents the unit commitment as an optimization problem where the total operational cost of a thermal portfolio is minimized. Its objective function (1) deals with every thermal units' production, shut-down, and start-up costs over the time span. The technical constraints that ensure a real operation are associated with this equation and gathered in the subsequent subsections.

$$min \left(\sum_{g \in G} \sum_{t \in T} c^{P}_{g,t} + c^{SD}_{g,t} + c^{SU}_{g,t} \right) \quad (1)$$

1) Production Constraints: Firstly, a characterization of the production cost is addressed. This feature is modeled through a linear equation where a linear variable cost acts as slope and a fixed cost as ordinate. The differentiation of a power output $p_{g,t}$ above the technical minimum $\underline{P_g}$, which sometimes helps the performance of the solver, is sustained in this methodology. The function can easily be replaced by a quadratic or piecewise approximation, representing the production cost better. For the sake of clarity, it has not been done, achieving a more precise comparison in the next section.

$$c^{P}_{g,t} = u_{g,t} C^{P-N}_{g} + (u_{g,t} \underline{P_g} + p_{g,t}) C^{P-M}_{g} \qquad \forall g, t \quad (2)$$

Secondly, generation limits are represented by just a single equation (3) when the thermal unit is not capable to start-up and shut-down in the same hourly period ($g \notin G^1$), which is described in [16], and by the equations (4) and (5) when TU_g is equal to 1 ($g \in G^1$).

$$\begin{aligned} p_{g,t} + r_{g,t} \leq u_{g,t}(\overline{P_g} - \underline{P_g}) - \sum_{s \in S} v_{g,t,s}(\overline{P_g} - SU_g) \\ - w_{g,t}(\overline{P_g} - SD_g) \qquad \forall g \notin G^1, t \end{aligned} \quad (3)$$

$$\begin{aligned} p_{g,t} + r_{g,t} \leq u_{g,t}(\overline{P_g} - \underline{P_g}) \\ - \sum_{s \in S} v_{g,t,s}(\overline{P_g} - SU_g) \qquad \forall g \in G^1, t \end{aligned} \quad (4)$$

$$\begin{aligned} p_{g,t} + r_{g,t} \leq u_{g,t}(\overline{P_g} - \underline{P_g}) \\ - w_{g,t}(\overline{P_g} - SD_g) \qquad \forall g \in G^1, t \end{aligned} \quad (5)$$

Thirdly, the ramping-limit constraints that guarantee the real operation of the thermal portfolio are modeled as follows. Note that the beginning of the time span is shown in Appendix A.

$$p_{g,t} + r_{g,t} - p_{g,t-1} \leq RU_g \qquad \forall g, t \in [2, T] \quad (6)$$

$$-p_{g,t} + p_{g,t-1} \leq RD_g \qquad \forall g, t \in [2, T] \quad (7)$$

2) Shut-down cost: This cost is closely related to the fuel consumption during the shut-down process. It does not change along the time horizon and is directly proportional to the fuel price. For this reason, it is always represented as a single step cost and frequently is obviated to simplify the problem. In this formulation, it is considered in order to increase the accuracy.

$$c^{SD}_{g,t} = w_{g,t} C^{SD}_{g} \qquad \forall g, t \quad (8)$$

3) Start-up cost: The start-up cost is often represented as a single step cost or barely differentiated by two steps (hot-cold) or even three (hot-warm-cold), [27]. As previously mentioned, stairwise functions have been used to model this feature, and start-up data provided in the literature commonly contemplate fast-flattering fuel-consumption curves. Regardless, piecewise linearizations bring an effective approach to deal with the high accuracy characterization of the start-up functions. In this case, different start-up phases can be represented, and the number of offline hours can be used to discern inside the same phase. This methodology can be leveraged to enhance the modeling detail of real fuel-consumption curves, shown in Fig. 1. There are several piecewise approximations. Fig. 3 illustrates a least-squares approximation composed by three linear segments.

The proposed formulation allows using any desired number of start-up segments. Each one is represented through a slope and an ordinate in the cost equation (9), except the last block. The coldest segment occurs when the curve is flat and consists of a fixed cost that does not change with an increase in offline hours. The value of the slope of this piece is zero.

$$c^{SU}_{g,t} = \sum_{s \in S} \left(v_{g,t,s} C^{SU-N}_{g,s} + h^{SU}_{g,t,s} C^{SU-M}_{g,s} \right) \quad \forall g, t \in T \quad (9)$$

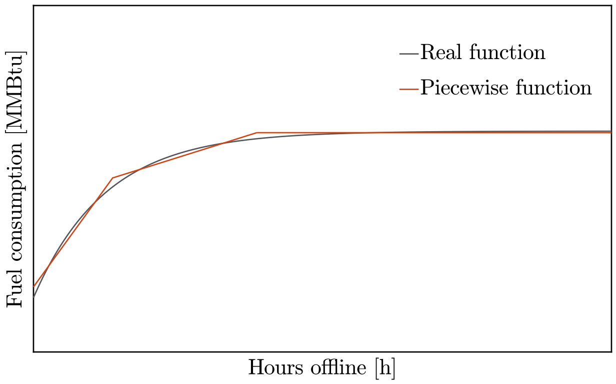

Fig. 3. Piecewise linearization of a real start-up fuel-consumption function. The piecewise function exposed in this figure shows three segments, related to a hot, warm and cold phase.

The number of hours that a thermal unit has been offline at each period is determined through the following constraints. If the unit is online, $h^{SD}_{g,t}$ is equal to zero. When it is shut-down, $h^{SD}_{g,t}$ begins to increase after every period in which the unit is de-committed. This variable can be defined as continuous, but it acquires an integer behavior due to its direct connection to binary variables. These equations use the parameter $\overline{H}$, a high value (e.g., the number of hours of a whole year) to guide the shut-down hours' counting. The latter objective of this Big-M functionality is not giving an integrality behavior to continuous variables that can be integer defined from the beginning, as is frequent in the literature. $\overline{H}$ is applied to maintain the linearity of the problem, while indirect relationships between decision variables are established. For the sake of simplicity, the initial conditions are stipulated in Appendix A.

$$h^{SD}_{g,t} \leq h^{SD}_{g,t-1} + 1 \qquad \forall g, t \in [2, T] \qquad (10)$$
$$h^{SD}_{g,t} - (1 - u_{g,t})\overline{H} \leq 0 \qquad \forall g, t \qquad (11)$$
$$h^{SD}_{g,t-1} + 1 \leq h^{SD}_{g,t} + u_{g,t}\overline{H} \qquad \forall g, t \in [2, T] \qquad (12)$$

Meanwhile, equation (13) determines the type of start-up s that corresponds to the offline hours $h^{SD}_{g,t}$, in accordance with the parameter $T^{SU}_{g,s}$. This constraint is not formulated for the coldest start-up phase because the logic equation (18) assures its assignment to v_{g,t,S_g} when it concerns.

$$v_{g,t,s} \leq 1 + \frac{T^{SU}_{g,s+1} - h^{SD}_{g,t-1}}{\overline{H}} \qquad \forall g, t, s \in [1, S_g) \qquad (13)$$

Finally, the next group of constraints take the value of $h^{SD}_{g,t}$ determined by (10-12) and assign it to the variable $h^{SU}_{g,t,s}$. This formulation guarantees that the offline hours $h^{SD}_{g,t}$ will be only assigned to the corresponding start-up segment. Hence, when a thermal unit starts-up, the variable $h^{SU}_{g,t,s}$ records the offline hours and accounts them in the start-up cost. It is important to note that these equations are not defined for the last segment because the curve is flat then and there is no reason to calculate the offline hours when the coldest start-up is activated.

$$h^{SD}_{g,t-1} - (1 - v_{g,t,s})\overline{H} \leq h^{SU}_{g,t,s} \qquad \forall g, t, s \in [1, S_g) \qquad (14)$$
$$h^{SU}_{g,t,s} \leq v_{g,t,s}\overline{H} \qquad \forall g, t, s \in [1, S_g) \qquad (15)$$

4) Balance constraints: These equations guarantee that the demand and spinning reserve of each period t of the time span T are plenty satisfied by the operation of the thermal portfolio. Some papers of the literature define the balance equation (16) as equality and add a non-served energy variable, penalized in the objective function. This term prevents infeasibilities when the demand curve does not fit the generation units' technical properties. However, it also avoids non-profitable productions when the operational cost exceeds the penalty. For that reason, following the security of supply energy policies, this equation has been transformed to a greater-equal inequality, suppressing the non-served energy term. Thus, the optimization will reduce the energy surplus to its minimum from an economic point of view.

$$D_t \leq \sum_{g \in G} u_{g,t}\underline{P_g} + p_{g,t} \qquad \forall t \qquad (16)$$
$$R_t \leq \sum_{g \in G} r_{g,t} \qquad \forall t \qquad (17)$$

5) Operational constraints: Firstly, equation (18) fixes the chronological relationships among the hourly periods, defining the logic between the commitments, start-ups, and shut-downs along the time span:

$$u_{g,t} - u_{g,t-1} = \sum_{s \in S} v_{g,t,s} - w_{g,t} \qquad \forall g, t \in [2, T] \qquad (18)$$

Hereafter, the subsequent constraints ensure to accomplish the minimum up/down hours that the generation units have to be online/offline due to operational flexibility limitations. As mentioned, initial conditions are specified in Appendix A.

$$\sum_{i=t-TU_g+1}^{t} \sum_{s \in S} v_{g,i,s} \leq u_{g,t} \qquad \forall g, t \in [TU_g, T] \qquad (19)$$
$$\sum_{i=t-TD_g+1}^{t} w_{g,i} \leq 1 - u_{g,t} \qquad \forall g, t \in [TD_g, T] \qquad (20)$$

B. Determination of the Start-up Parameters

The start-up cost is directly related to the fuel consumption during this operation, which is higher as the cooling processes draw on. The above-mentioned mathematical behavior of these curves (21) is difficult to manage in MILP formulations. Thus, these fuel consumption functions have to be linearized through the application of piecewise approximations or stairwise representations.

$$F^{SU-REAL}_{g,t} = A_g - B_g \, exp\left(\frac{-C_g}{\tau}\right) \qquad (21)$$

The start-up representation by a stairwise formulation where each step entails a fixed cost has been widely employed in the literature. It shows computationally efficient results, especially when a few steps can accurately model fast-flattering curves. However, real data of combined cycle gas turbines, like those presented in Appendix B and illustrated in Fig. 1, reveal longer flattering curves. Consequently, when the time spans consider more than the typical one day to one week on an hourly basis, piecewise linearizations gain ground as exposed in Section III.

TABLE I
TECHNICAL DATA OF THE THERMAL UNITS.

Thermal Unit	F_g^{P-M} [MMBtu/MWh]	F_g^{P-N} [MMBtu/h]	F_g^{SD} [MMBtu]	$F_{g,hot}^{SU-M}$ [MMBtu/h]	$F_{g,warm}^{SU-M}$ [MMBtu/h]	$F_{g,hot}^{SU-N}$ [MMBtu]	$F_{g,warm}^{SU-N}$ [MMBtu]	$F_{g,cold}^{SU-N}$ [MMBtu]	$\overline{P_g}$ [MW]	$\underline{P_g}$ [MW]	RD_g [MW/h]	RU_g [MW/h]	SD_g [MW]	SU_g [MW]	TD_g [h]	TU_g [h]	$T_{g,warm}^{SU}$ [h]	$T_{g,cold}^{SU}$ [h]
Unit A	6.6	300	1100	392.3	77.9	1517.4	3545.3	4899.2	412	157	215	215	157	157	7	7	7	18
Unit B	6.2	460	1100	42.2	9.3	1142.7	3622.2	5603.3	390	135	200	200	135	135	5	5	76	213
Unit C	5.4	820	1900	326.3	9.0	768.6	5280.0	8696.9	856	285	425	425	285	285	11	11	18	50
Unit D	6.4	320	1100	18.5	4.0	2178.3	2999.7	3648.3	402	112	200	200	112	112	6	6	57	161
Unit E	6.8	280	1200	12.6	2.8	2691.1	3505.9	4169.3	413	157	215	215	157	157	7	7	84	236
Unit F	6.2	360	1200	31.9	7.3	1846.3	4298.0	6329.5	427	163	225	225	163	163	8	8	100	279
Unit G	5.6	740	1900	25.9	5.8	4821.5	6578.8	7995.8	796	225	385	385	225	225	10	10	88	246

Piecewise linearizations can be addressed through different approaches, like upper, lower, and mixed linear interpolation or least-squares approximations. An analysis of the performance of a linear-interpolant technique and an orthogonal-projection methodology is made in [35], where it is determined that the least-squares approximation is the most convenient method for minimizing the error. Accordingly, the least-squares piecewise linearization proposed in Appendix C is applied. This approach presents the approximation as an optimization problem.

Once the slopes, ordinates, and segment intervals are determined for each thermal unit, the consumption parameters are multiplied by the corresponding fuel price:

$$C_g^{SU-M} = F_g^{SU-M} C_g^F \tag{22}$$

$$C_g^{SU-N} = F_g^{SU-N} C_g^F \tag{23}$$

Similarly, the one-step shut-down and the linear production consumption represented in this paper are multiplied by fuel prices before entering the formulation. An advantage of using fuel consumption instead of direct costs is the easy parameter-reassignment to optimize the operation when markets change.

$$C_g^{SD} = F_g^{SD} C_g^F \tag{24}$$

$$C_g^{P-M} = F_g^{P-M} C_g^F \tag{25}$$

$$C_g^{P-N} = F_g^{P-N} C_g^F \tag{26}$$

Finally, in order to establish a proper comparison benchmark to analyze the computational performance of this formulation against the linearizations shown in the literature, both start-up representations must report the same mean absolute percentage error (MAPE). Hence, the step-aggregation method proposed in [36] and enhanced in Appendix D is employed to obtain a stairwise function from the fuel consumption curves gathered in Appendix B.

III. CASE STUDIES AND COMPUTATIONAL PERFORMANCE

This paper proposes a computationally efficient formulation of the unit commitment problem to improve the representation accuracy of the start-up costs. Furthermore, to demonstrate its validity, resolution performance is compared against the robust methodology of [16] at several medium-term case studies.

It is important to note that the new proposal takes equations from [16] to model those technical features unrelated to start-up implications. Thus, a rigorous analysis of the representation of the start-up details is performed without the interference of any other modeling difference, making a fairer comparison.

A. Description of the Thermal Portfolio

The operation of a thermal generation portfolio composed of 7 combined cycle gas turbines placed in the Iberian Electricity Market is optimized at the case studies presented in this paper. Their technical parameters are gathered in Table I, except for the start-up fuel consumption curves, whose data are provided in Appendix B. Results of applying the least-squares piecewise linearization presented in Appendix C also appear in Table I. Three segments are assigned at the linearization to distinguish between the hot, warm, and cold start-up phases. Accordingly, the case studies closely represent the operation of real systems.

B. Description of the Medium-Term Horizons

The technical information of the thermal portfolio reveals a long-flattering behavior when start-up curves are represented. This fact, together with the current market trends that require an increase in the number of start-up and shut-down processes for the combined cycle gas turbines, demands the employment of time spans longer than 24 hours or one week.

With this aim, medium-term horizons that comprise a whole month on an hourly basis were selected for each case study, allowing an in-depth analysis of operational decisions without incurring an excessive simplification of the start-up processes. Every month in a year is evaluated to discern seasonality and study different market trends, like fuel prices or power-demand curves. Regarding this concern, 2020 is selected to build the case studies. This year constitutes a good characterization for the generation technologies of the Iberian Electricity Market, and reflects a rational behavior in spot and future commodity markets. Given the location of the units, the monthly average cost in the Iberian Gas Market (MIBGAS) is utilized as fuel cost [37]. These data are shown in Table II.

C. Description of the Power-Demand Curves

As described in Section I, many case studies proposed in the literature repeat a daily hourly power-demand profile to extend the time span. Nevertheless, it does not represent medium-term horizons properly. Hence, the historical data corresponding to the hourly productions of all the combined cycle gas turbines placed in the Iberian Electricity Market along 2020 [38], have been utilized as power-demand input parameters.

This information needs to be scaled to the maximum power output of the generation portfolio before its utilization in case studies. Firstly, the 2020 profile is split by month. Afterwards, the higher monthly-production peak is adjusted to reach 95% of the portfolio's maximum capacity, as Equation (27) defines:

$$D_t = \frac{Gas\ Fired\ Production_{\ t}}{max\{Gas\ Fired\ Production_{\ t}\}} 0.95 \sum_{g \in G} \overline{P_g} \quad (27)$$

Thereafter, to avoid infeasible demands that require a lower production than the minimum power output of the least unit, a replacement by zero-demand hours is done in this situation. Fig. 2 illustrates the transformation of an accurate gas-fired production profile into a power-demand curve for a combined cycle gas turbine portfolio in a medium-term horizon.

Finally, the spinning reserve requirement is set as 5% of the power-demand for each period t, as frequent in the literature.

D. Numerical Results of the Case Studies

As previously said, the mathematical formulation proposed in this paper is compared against the methodology presented in [16]. With the aim of exhibiting an accurate analysis, real start-up functions are linearized through the approaches gathered in Appendices C and D, reaching the same MAPE after applying both approximations.

Likewise, medium-term horizons are established in the case studies to deal with the increasing importance of start-up and shut-down processes in the current electricity markets, and real variability in power-demand curves is characterized to enhance the scope of the representations. In accordance, twelve case studies are evaluated. Each one corresponds to an entire month of 2020 on an hourly basis.

They have been run in a computer Intel Core i7-8700 @3.20 GHz with 12 logical processors and 32 GB of installed RAM memory running 64-bit Windows 10 Pro, and solved with the commercial solver Gurobi 9.5 [39] under GAMS [40]. Due to the big size of these problems, an aggressive presolve is set at Gurobi options, and an optimality gap of 1.0% is chosen. The run times are shown in Table II. It can be appreciated that the piecewise formulation (PWF) achieves a ∼50% reduction of the run time required by the stairwise formulation (SWF), demonstrating its efficiency and its ability to manage a higher detail representation.

The problem sizes with each formulation are shown in Table III. After the application of the presolver (AP), PWF reports ∼42% constraints (#CT), ∼260% continuous variables (#CV), ∼19% binary variables (#BV), and ∼87% non-zeroes (#NZ) when comparing to SWF. These differences are greater before the presolve (BP). Nevertheless, the SWF dimensionality alerts the optimization processes, which notably dedicate more time to generate the MILP models and spend higher computational resources at the presolve step. Therefore, tighter polytopes and lower initial gaps are obtained with SWF.

Before the presolve phase, the dimensions of both problems can be determined through the following equations:

- PWF constraints: according to the formulation presented in Section II-A, the constraint quantity of each PWF case study can be calculated as follows.

$$\#CT_{\text{PWF}} = \sum_{t \in T} \left(2 + \sum_{g \notin G^1} \left(9 + \sum_{s}^{S-1} 3 \right) + \sum_{g \in G^1} \left(10 + \sum_{s}^{S-1} 3 \right) \right) - \sum_{g \in G} \left(TU_g + TD_g - 2 - TU_g^R - TD_g^R \right) \quad (28)$$

TABLE II
CASE STUDIES, FUEL PRICES, AND RUN TIME COMPARISONS.

Case	C_g^F [\$/MMBtu]	Run Time PWF [s]	Run Time SWF [s]	Run Time PWF / Run Time SWF
Jan.	3.864	552.738	1334.539	0.414
Feb.	3.138	271.364	393.331	0.690
Mar.	2.776	821.474	1842.149	0.446
Apr.	2.381	2228.233	5974.488	0.373
May	1.704	554.767	1665.991	0.333
June	2.118	229.196	712.867	0.322
July	2.160	433.626	636.259	0.682
Aug.	3.186	203.761	397.425	0.513
Sept.	3.909	210.033	771.245	0.272
Oct.	4.639	932.582	1481.440	0.630
Nov.	5.029	647.554	803.245	0.806
Dec.	6.483	1204.830	3003.471	0.401

- PWF continuous variables: this methodology defines $p_{g,t}$, $r_{g,t}$, $h_{g,t}^{SD}$, and $h_{g,t,s}^{SU}$ as continuous. It is important to note that the last start-up segment is flat, avoiding $h_{g,t,cold}^{SU}$.

$$\#CV_{PWF} = \sum_{t \in T} \sum_{g \in G} \left(3 + \sum_{s}^{S-1} 1 \right) \quad (29)$$

- PWF binary variables: the quantity of binary variables is easily determined by the sum of $u_{g,t}$, $v_{g,t,s}$, and $w_{g,t}$.

$$\#BV_{PWF} = \sum_{t \in T} \sum_{g \in G} \left(2 + \sum_{s \in S} 1 \right) \quad (30)$$

- SWF constraints: according to the formulation presented in [16], the constraint quantity of each SWF case study can be determined as follows.

$$\#CT_{\text{SWF}} = \sum_{t \in T} \left(2 + \sum_{g \notin G^1} \left(8 + \sum_{s}^{S-1} 1 \right) + \sum_{g \in G^1} \left(9 + \sum_{s}^{S-1} 1 \right) \right) - \sum_{g \in G} \left(TU_g + TD_g - 2 - TU_g^R - TD_g^R \right) - \sum_{g \in G} \sum_{s}^{S-1} \left(T_{s+1}^{SU} - TD_g^0 \right) \quad (31)$$

- SWF continuous variables: [16] just defines $p_{g,t}$ and $r_{g,t}$.

$$\#CV_{SWF} = \sum_{t \in T} \sum_{g \in G} 2 \quad (32)$$

- SWF binary variables: the quantity of binary variables is given by $u_{g,t}$, $v_{g,t}$, $w_{g,t}$, and $\delta_{g,s,t}$, which models a start-up decision for each step of the stairwise linearization.

$$\#BV_{SWF} = \sum_{t \in T} \sum_{g \in G} \left(3 + \sum_{s \in S} 1 \right) \quad (33)$$

It is important to highlight that PWF achieves a satisfactory representation of the start-up curves with only three segments s. Meanwhile, SWF needs up to 38 start-up steps s to reach the same accuracy (Appendix D). This fact implies a considerable difference between both problem dimensions regarding binary variables, which entail a high computational burden.

Consequently, the SWF problem size is bigger than PWF in spite of the better performance of the presolve. In accordance, a clear difference is appreciated when their relaxed MILPs are solved. PWF needs ∼10 seconds to optimize its corresponding linear programming problem, while SWF takes ∼200 seconds.

TABLE III
CASE STUDY PROBLEM SIZES AND PERFORMANCE OF THE PRESOLVER IN BOTH FORMULATIONS.

Case	Gen. MIP Model [s]		#CT - BP (10^3)		#CV - BP (10^3)		#BV - BP (10^3)		Presolve Time [s]		#CT - AP (10^3)		#CV - AP (10^3)		#BV - AP (10^3)		#NZ - AP (10^3)		Initial Opt. Gap [%]	
	PWF	SWF	PWF	SWF	PWF	SWF	PWF	SWF	PWF	SWF	PWF	SWF	PWF	SWF	PWF	SWF	PWF	SWF	PWF	SWF
Jan.	8.8	194.0	79.5	180.7	26.0	10.4	26.0	213.5	8.7	52.5	60.7	140.7	24.4	9.8	24.3	126.9	1132.8	1249.9	28.3	26.9
Feb.	7.4	157.1	74.4	168.4	24.4	9.7	24.4	199.8	100.7	52.9	51.9	121.1	21.1	8.1	21.5	109.5	934.7	1026.5	25.6	19.5
Mar.	8.7	190.6	79.5	180.7	26.0	10.4	26.0	213.5	7.1	50.6	59.8	143.9	23.9	8.9	25.0	132.1	1121.6	1282.6	45.8	40.8
Apr.	7.9	173.0	76.9	174.5	25.2	10.1	25.2	206.6	7.6	48.8	58.4	142.4	22.5	7.7	24.6	132.2	1040.9	1284.8	61.1	52.8
May	8.5	204.6	79.5	180.7	26.0	10.4	26.0	213.5	14.4	47.7	54.6	143.0	20.1	6.8	23.5	133.5	985.2	1297.7	54.8	38.4
June	8.0	174.6	76.9	174.5	25.2	10.1	25.2	206.6	7.4	51.0	57.0	131.3	23.6	10.1	22.4	116.7	1034.1	1134.2	18.8	12.7
July	8.4	194.8	79.5	180.7	26.0	10.4	26.0	213.5	77.7	54.7	56.7	127.3	23.6	10.3	22.0	112.1	976.5	1074.7	8.7	8.4
Aug.	8.5	196.5	79.5	180.7	26.0	10.4	26.0	213.5	26.1	51.4	50.8	108.5	21.9	10.4	18.9	92.4	846.0	879.7	7.0	6.9
Sept.	7.9	178.3	76.9	174.5	25.2	10.1	25.2	206.6	9.2	54.9	52.3	121.3	21.9	10.0	20.5	106.6	924.3	1056.3	13.5	10.0
Oct.	8.7	191.8	79.5	180.7	26.0	10.4	26.0	213.5	7.8	52.0	58.1	137.4	22.6	8.2	24.0	125.7	1060.7	1214.0	34.7	25.2
Nov.	8.1	166.6	76.9	174.5	25.2	10.1	25.2	206.6	8.1	51.5	57.1	134.3	22.4	8.4	23.2	122.2	1006.9	1173.9	30.7	23.3
Dec.	8.6	190.7	79.5	180.7	26.0	10.4	26.0	213.5	6.9	50.7	58.1	142.9	21.9	6.8	25.0	133.0	1094.8	1285.3	48.7	34.4

This fact facilitates the exploration of the enumeration tree by the PWF, visiting more nodes, establishing additional cuts, and using extra heuristics. The resolution performances are shown in Table IV. Most of the relaxed run time (∼83% in PWF and ∼94% in SWF) is utilized at the model generation, manifesting the relevance of the greater compactness achieved with PWF.

There is no clear conclusion regarding the simplex iterations to reach the optimal solution. However, it is denoted that PWF addresses higher iterations per second than SWF. Thereupon, it can be concluded that mapping the feasible region is easier with PWF. Finally, it is important to note that SWF tends to reach a lower final optimality gap, but this parameter would be humbled if run time limits were imposed. Moreover, the better integrality gaps obtained with SWF reveal its greater tightness. Despite that, PWF accomplishes an improved tight & compact trade-off from a computational efficiency perspective.

Thereafter, it is interesting to analyze the accuracy of those results returned by the solver after optimizing both problems. Given the application of different linearization methods in the formulations, it is quite probable that they will not reach the exact optimal solution. Nonetheless, quality parameters can be established to discern the truthfulness of the start-up modeling.

The inferred decision variables after the resolution are used to calculate their corresponding real start-up costs through the Exponential Functions (EF) described in Appendix B. Hence, the closer the approaches are to the EF, the more accurate the start-up modeling is. This comparison is exposed in Table V. As can be observed, it is frequent to reduce the start-up costs. That is a rational behavior according to the cost-minimization definition of the problem. However, the total cost is oversized in some case studies. This situation is related to non-flexible operational constraints. In that case, the solver is not capable of choosing those segments/steps of the linearizations that reflect an underestimated cost.

When both formulations are contrasted, an overall start-up accuracy of 91.6% is achieved with the PWF, while SWF only reaches 74.6%. Even though both linearization methods have assumed the same MAPE (Appendix D), the PWF manifests a better performance. This concern can be explained by the fitter time dependency accomplished with piecewise functions. With them, each offline hour is differently measured in the start-up cost of the objective function.

In contrast, the offline hours are sometimes aggregated at the step definitions in stairwise linearizations. The solver leverages this consideration as a block to compute the minimal possible start-up cost, playing with the offline hours to approximate the last value of the intervals. This step aggregation is a significant drawback of stairwise formulations because providing a clear hourly distinction requires too many steps and entails computationally demanding resolution processes. For that reason, the proposed piecewise functions to accurately model start-ups are validated as a computationally efficient methodology.

TABLE IV
COMPUTATIONAL PERFORMANCE OF THE SOLVER WHEN OPTIMIZING THE CASE STUDIES WITH BOTH FORMULATIONS.

Case	Nodes		Cuts		Heuristics		Simplex Iterations		Simplex Iterations per second		Nodes per second		Relaxed MIP Run Time [s]		Final Opt. Gap [%]		Integrality Gap [%]	
	PWF	SWF	PWF	SWF	PWF	SWF	PWF	SWF	PWF	SWF	PWF	SWF	PWF	SWF	PWF	SWF	PWF	SWF
Jan.	3192	1248	10059	0	38	39	691490	715173	1251	628	5.775	0.935	10.511	218.961	0.952	0.973	3.680	3.141
Feb.	3058	1	9357	3114	28	13	325336	140948	1199	600	11.269	0.003	9.001	181.084	0.999	0.911	3.235	2.807
Mar.	3351	897	10862	4273	39	25	938198	763794	1142	463	4.079	0.487	10.461	216.283	0.993	0.968	5.845	5.288
Apr.	7233	1541	10499	4239	77	30	3322514	2079339	1491	359	3.246	0.258	9.791	194.616	0.993	0.841	9.241	8.185
May	2109	697	7263	3065	24	23	705981	641866	1273	440	3.802	0.418	10.297	222.637	0.996	0.992	5.965	5.336
June	711	247	10391	4169	21	32	307149	287852	1340	536	3.102	0.346	9.761	196.191	0.835	0.926	2.355	2.121
July	3476	856	9464	3355	21	26	423812	336271	977	764	8.016	1.345	10.365	214.005	0.980	0.949	1.998	1.814
Aug.	1588	1	8289	3413	16	13	312987	154717	1536	775	7.793	0.003	10.085	213.293	0.595	0.874	1.617	1.771
Sept.	3759	703	10299	3871	53	33	466251	499006	2220	843	17.897	0.912	9.659	202.246	0.981	0.970	2.232	2.161
Oct.	3469	1156	9303	3681	52	38	832666	698839	893	542	3.720	0.780	10.567	216.312	0.995	0.937	4.442	3.952
Nov.	3812	398	11066	3753	52	26	728339	356489	1125	561	5.887	0.495	9.846	191.333	0.994	0.969	3.945	3.216
Dec.	3910	1733	9192	3385	80	37	912872	1281967	758	456	3.245	0.577	10.628	222.177	0.992	0.963	6.120	5.298

IV. Conclusions

Ongoing trends in electricity markets demand an increase in the detail of energy models. The demand-variability increment caused by the high penetration of non-dispatchable renewable energy resources positions fast-ramping thermal units as a key vector towards a clean energy transition.

Accordingly, an adaptation of their operation is required to meet the new demand-profiles, starting-up, and shutting-down with a greater frequency for producing during less time. With this regard, the unit commitment problem is a powerful tool to optimize the operation of generation units.

Nevertheless, the modeling accuracy of these operations is frequently oversimplified in the literature. Low detail in start-up costs and unrealistic power-demand curves are still utilized nowadays to reduce the computational burden of this complex problem. Hence, a computationally efficient formulation where an accurate representation of the start-up processes is allowed is proposed in this paper.

This approach replaces the conventional stairwise modeling of the unit commitment's start-up costs with a more qualified piecewise formulation, establishing an original set of equations that ensure keeping the linearity of the problem. The validity of this methodology has been demonstrated by its comparison to one of the most renowned formulations of the literature. In turn, several case studies that characterize the reality of current power systems are presented.

The operation of a thermal portfolio composed of combined cycle gas turbines is optimized in medium-term case studies on an hourly basis, where the importance of properly representing start-up and shut-down processes is manifested. If desired, this methodology can also be utilized as a basis to model any other important feature of the operation of electricity markets, such as considerating optimal power flow constraints, configuration transitions, dynamic ramping, or incorporating uncertainty.

Furthermore, real demand curves that boost computational requirements are employed. To conclude, numerical results are exposed to clearly illustrate the effectiveness of the resolution performance achieved with the proposed formulation. Utilizing this methodology provides a notable reduction of the run time, allowing a higher detail representation in the unit commitment. Moreover, the start-up costs determined at the optimization are closer to those calculated with real exponential curves, proving the superior accuracy of the proposed MILP formulation.

TABLE V
Comparison of the Linearized Start-Up Costs to Their Corresponding Values Determined by Exponential Curves.

Case	PWF [k$]	$\mathrm{EF_{PWF}}$ [k$]	$\frac{\mathrm{PWF}}{\mathrm{EF_{PWF}}}$	SWF [k$]	$\mathrm{EF_{SWF}}$ [k$]	$\frac{\mathrm{SWF}}{\mathrm{EF_{SWF}}}$
Jan.	606.374	769.197	0.788	585.714	759.367	0.771
Feb.	479.237	607.112	0.789	497.718	746.049	0.667
Mar.	317.564	229.352	1.385	344.664	347.185	0.993
Apr.	290.178	350.128	0.829	281.097	377.914	0.744
May	190.657	127.989	1.490	197.651	171.183	1.155
June	245.305	305.863	0.802	266.509	397.000	0.671
July	244.337	328.902	0.743	251.941	470.006	0.536
Aug.	344.739	525.280	0.656	404.640	790.722	0.512
Sept.	448.824	489.091	0.918	498.744	721.140	0.692
Oct.	799.672	857.763	0.932	790.249	990.018	0.798
Nov.	945.262	1097.767	0.861	932.422	1268.829	0.735
Dec.	1148.967	1442.916	0.796	1095.890	1609.232	0.681

TABLE VI
Initial Conditions of the Thermal Units.

Thermal Unit	P_g^0 [MW]	TD_g^0 [h]	TU_g^0 [h]	U_g^0
Unit A	314	0	7	1
Unit B	270	0	5	1
Unit C	570	0	11	1
Unit D	224	0	6	1
Unit E	314	0	7	1
Unit F	326	0	8	1
Unit G	450	0	10	1

Appendix

A. Initial Conditions

The optimization behavior at the beginning of the time span is defined by the initial conditions of the thermal units. Hence, the following parameters and equations determine the correct operation modeling along the first periods of the horizon.

P_g^0 Power output of unit g in the first period t [MW].
TD_g^0 Offline hours of unit g in the first period t [h].
TD_g^R Number of hours that unit g must remain offline [h].
TU_g^0 Online hours of unit g in the first period t [h].
TU_g^R Number of hours that unit g must remain online [h].
U_g^0 Commitment status of unit g in the first period t.

1) Initial minimum up and down times: At the initial time period of the horizon, the number of online/offline hours that each thermal unit accounts need to be processed to determine the interval within their commitment status should not change. These parameters are calculated before the optimization:

$$TU_g^R = max\{0, (TU_g - TU_g^0)U_g^0\} \qquad \forall g \qquad (34)$$

$$TD_g^R = max\{0, (TD_g - TD_g^0)(1 - U_g^0)\} \qquad \forall g \qquad (35)$$

Afterwards, they are employed to guarantee the invariability of the commitment status and keep the logic in chronological relationships between time periods.

$$u_{g,t} = U_g^0 \qquad \forall g, t \in [1, TU_g^R + TD_g^R] \qquad (36)$$

$$u_{g,t} - U_g^0 = \sum_{s \in S} v_{g,t,s} - w_{g,t} \qquad \forall g, t \in [1, 2) \qquad (37)$$

2) Initial shut-down hours: Accordingly to the initial status of the minimum down time, the value of the number of hours that each unit has been offline in the first period of the horizon is determined through the following equations.

$$h_{g,t}^{SD} \leq TD_g^0 + 1 \qquad \forall g, t \in [1, 2) \qquad (38)$$

$$TD_g^0 + 1 \leq h_{g,t}^{SD} + u_{g,t}\overline{H} \qquad \forall g, t \in [1, 2) \qquad (39)$$

3) Initial ramping limits: The initial ramping capability of the thermal units is characterized by their power output at the beginning of the represented time span.

$$p_{g,t} + r_{g,t} - (P_g^0 - U_g^0 \underline{P_g}) \leq RU_g \qquad \forall g, t \in [1, 2) \qquad (40)$$

$$-p_{g,t} + (P_g^0 - U_g^0 \underline{P_g}) \leq RD_g \qquad \forall g, t \in [1, 2) \qquad (41)$$

The technical information about the initial conditions of the thermal portfolio presented in this paper is shown in Table VI.

TABLE VII
START-UP FUEL-CONSUMPTION PARAMETERS.

Thermal Unit	A_g [MMBtu]	B_g [MMBtu]	C_g [h]
Unit A	4900	3820	5
Unit B	5630	4830	67
Unit C	8705	8640	15
Unit D	3654	1590	50
Unit E	4180	1600	75
Unit F	6374	4860	90
Unit G	8020	3440	78

B. Fuel-Consumption Function Data

The start-up cost of a thermal unit is directly related to the fuel consumption during this process, which is mathematically described through an exponential function of the offline time prior to the start-up. This behavior is defined in equation (21), whose parameters for the generation portfolio presented in this paper are gathered in Table VII.

These curves are illustrated in Fig. 1, where it is appreciated an increase in fuel consumption as the facility cools down and reaches ambient temperature. Nomenclature is exposed below. Note that there is a slight difference between the value reported by equation (21) and those provided by the piecewise and the stairwise linearizations, which is measured through the MAPE reached along the whole hours of each time span.

$F_{g,t}^{SU-REAL}$	Real start-up fuel-consumption [MMBtu].
A_g	First parameter of the function [MMBtu].
B_g	Second parameter of the function [MMBtu].
C_g	Third parameter of the function [h].
τ	Offline hours before the start-up [h].

C. Piecewise Linearization Method

According to [35], the best procedure to perform a piecewise linearization is utilizing a least-squares approximation. For the sake of simplicity, a numerical calculation is proposed in this paper instead of appealing to analytical approaches. It consists of an optimization problem. The fuel consumption information determined with equation (21) is introduced as a target in the objective function (42), where a least-squares minimization is applied. Constraints (43)-(50) establish the linearization rules, in which y_t, m_t and n_t are defined as positive continuous variables, $\widetilde{m_t}$ and $\widetilde{n_t}$ as binary, and N^{PW} as a parameter.

y_t	Forecasted fuel-consumption in period t [MMBtu].
m_t	Slope value in period t [MMBtu/h].
$\widetilde{m_t}$	Variation of the slope value in period t.
n_t	Ordinate value in period t [MMBtu].
$\widetilde{n_t}$	Variation of the ordinate value in period t.
N^{PW}	Number of segments in the piecewise linearization.

The following method is suitable for obtaining a piecewise linearization of any exponentially growth-decreasing function, like a start-up fuel consumption curve. Constraints are specifically formulated taking advantage of their previously-known behavior.

- Equation 43 assures that the calculated segments of the piecewise function are linear.
- Equation 44 assures a decreasing behavior for the slopes of the segments that integrate the piecewise function.
- Equation 45 assures an increasing trend for the ordinates of the segments that integrate the piecewise function.
- Equation 46 assures that the slope-variation identifier $\widetilde{m_t}$ registers a value of 1 if the slope changes from one hour to another.
- Equation 47 assures that the ordinate-variation identifier $\widetilde{n_t}$ registers a value of 1 if the ordinate changes from one hour to another.
- Equation 48 assures that an ordinate variation in period t entails a slope variation in the same period.
- Equation 49 establishes a logical constraint. The number of slope variations and ordinate variations is lower than the number of segments minus one.
- Equation 50 assures that the slope of the last segment of the piecewise function is equal to zero, according to the curve flattening when the coldest start-up is reached.

$$min \left(\sum_{t \in T} \left(F_t^{SU-REAL} - y_t \right)^2 \right) \tag{42}$$

subject to

$$y_t = n_t + m_t \sum_{i=1}^{t} i \qquad \forall t \tag{43}$$

$$m_t \leq m_{t-1} \qquad \forall t \in [2, T] \tag{44}$$

$$n_{t-1} \leq n_t \qquad \forall t \in [2, T] \tag{45}$$

$$\frac{m_{t-1} - m_t}{\overline{H}} \leq \widetilde{m_t} \qquad \forall t \in [2, T] \tag{46}$$

$$\frac{n_t - n_{t-1}}{\overline{H}} \leq \widetilde{n_t} \qquad \forall t \in [2, T] \tag{47}$$

$$\widetilde{n_t} - \widetilde{m_t} \leq 0 \qquad \forall t \tag{48}$$

$$\sum_{t \in T} \widetilde{m_t} \leq N^{PW} - 1 \qquad \forall t \tag{49}$$

$$m_t \leq \overline{H} \left(N^{PW} - 1 - \sum_{i=1}^{t} \widetilde{m_i} \right) \qquad \forall t \tag{50}$$

In order to help the solver to find the optimal solution of this MIQCP problem, the value or the ordinate during the first time period is initialized as follows, $n_{t1} = F_{t1}^{SU-REAL}$. This initialization is not an equation that belongs to the formulation. It is just a previous assignment, and its accomplishment is not mandatory. Actually, when the results of the optimization are obtained, the real value of n_{t1} differs from this starting point.

D. Stairwise Aggregation Method

When a stairwise approximation is employed to represent an exponentially growth-decreasing curve in the unit commitment literature, it is not common to mention what methodology has been used to determine the corresponding steps. Nevertheless, an aggregation algorithm is described in [36]. It calculates the most representative aggregated steps according to a predefined error tolerance. Its nomenclature is exposed below:

$F_{g,s}^{SU}$	Forecasted fuel-consumption for start-up s [MMBtu].
I_g	Error tolerance for the start-up step aggregation [%].
t_a	Initial time period for a start-up step [h].
t_b	Final time period for a start-up step [h].

Algorithm 1: Start-up step aggregation methodology.

$g \leftarrow thermalUnit$;
$t_a, t_b, s \leftarrow 1$;
while $t_a \leq |T|$ **do**
  while $t_b + 1 \leq |T| \wedge error\,(g, t_a, t_b + 1) \leq I_g$ **do**
    $t_b \leftarrow t_b + 1$;
  $T_{g,s}^{SU} \leftarrow t_a$;
  if $t_a = t_b$ **then**
    $F_{g,s}^{SU} \leftarrow F_{g,t_a}^{SU-REAL}$;
    $t_a \leftarrow t_a + 1$;
  else
    $F_{g,s}^{SU} \leftarrow step\,(g, t_a, t_b)$;
    $t_a \leftarrow t_b + 1$;
  $t_b \leftarrow t_b + 1$;
  $s \leftarrow s + 1$;

This paper proposes a modified version of this algorithm in order to not aggregate hourly steps when the tolerance is not satisfied. In that case, the real fuel consumption data are kept during the corresponding time period. On the other side, when the tolerance is accomplished, the iterative algorithm begins a step aggregation until this error limit is reached. Then, optimal step values are calculated as it is done in [36], where equations (51) and (52) are presented.

$$error\,(g, t_a, t_b) = \frac{F_{g,t_b}^{SU-REAL} - F_{g,t_a}^{SU-REAL}}{F_{g,t_b}^{SU-REAL} + F_{g,t_a}^{SU-REAL}} \quad \forall g, t \quad (51)$$

$$step\,(g, t_a, t_b) = \frac{2F_{g,t_b}^{SU-REAL} F_{g,t_a}^{SU-REAL}}{F_{g,t_b}^{SU-REAL} + F_{g,t_a}^{SU-REAL}} \quad \forall g, t \quad (52)$$

Algorithm 1 together with equations (51) and (52) are used to determine the necessary start-up parameters for the stairwise linearization of [16]. At this stage, the formulation proposed in this paper can be compared to [16]. As previously mentioned, both units' start-up linearizations must report the same MAPE to perform an accurate comparison.

$$MAPE_g = \frac{1}{T} \sum_{t \in T} \left| \frac{F_{g,t}^{SU-REAL} - F_{g,t}^{*}}{F_{g,t}^{SU-REAL}} \right| \quad (53)$$

Therefore, I_g data are tested in the algorithm 1 and equation (53) until reaching the same modeling detail that is obtained in the three-segment piecewise linearization. Note that $F_{g,t}^{*}$ is equivalent to y_t in the piecewise linearization, and to the $F_{g,s}^{SU}$ that corresponds to each period t of the evaluated monthly time span for the stairwise linearization. It is exposed in Table VIII.

TABLE VIII
MEAN ABSOLUTE PERCENTAGE ERRORS MADE IN THE LINEARIZATIONS.

Thermal Unit	$MAPE_g^{PW}$	I_g [%]	# Steps	$MAPE_g^{SW}$
Unit A	0.001	0.250	21	0.001
Unit B	0.011	2.150	36	0.011
Unit C	0.003	0.775	35	0.005
Unit D	0.004	0.700	33	0.004
Unit E	0.004	0.550	38	0.003
Unit F	0.011	2.750	24	0.011
Unit G	0.005	1.000	26	0.005

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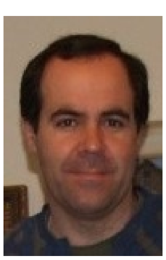

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