Guaranteeing the Provision of Primary Frequency Control Services by Distributed Generation

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***Abstract*—In this work, a tool for the evacuation of Primary Frequency Control (PFC) service by generators connected to the distribution network is proposed. To achieve this goal, a linear AC Optimal Power Flow (OPF) model with a minimization objective and generation re-dispatch constraint is developed to guarantee the provision of the control service. Tests are carried out on an 8-bus system and a real High Voltage (HV) distribution grid. The obtained results revealed that the algorithm can guarantee the provision of the required PFC service provision by re-dispatching the connected generators while at the same time satisfying other operation and network constraints.**

***Index Terms*—Ancillary Service Provision, Linear AC Power flow, Primary Frequency control, Real-Time Redispatch.**

I. Introduction

In real-time operation of power systems, it is always required to maintain frequency and voltage within certain bounds. These bounds are centered around given nominal values. In order to ensure that both frequency and voltage are within bounds, necessary control measures have to be put in place during power system operation. For frequency, generally there are three control levels: primary, secondary and tertiary control. The focus in this work is on primary frequency control which is the fastest among the three control levels. Other frequency control services exist such as Fast Frequency Response (FFR), but their delivery period is usually much shorter than the one of PFC.

In power systems dominated by synchronous machines, the first derivative of the frequency depends on the active power unbalance and the system inertia. Thus, after an active power unbalance, the primary control limits the frequency deviation by increasing or decreasing the active power generation to re-establish the active power balance. Although this control is typically carried out by means of the turbine-governor system of synchronous generation, recent studies demonstrate that non-synchronous generation, which is mainly connected to the distribution network, also has the ability to emulate Inertia and Primary Frequency Control (I+PFC) service provision [1], [2].

Another technology that is rapidly coming into play in the I+PFC is Battery Energy Storage System (BESS), whose installation allows for deferral of investments for grid reinforcements [3], and enables the operation of distribution grids closer to their capacity limits [4]. The operation of any grid is limited by (i) the acceptable voltage ranges and voltage drops and (ii) the active power flow capacity of network branches. Branch flows close to the capacity limit might affect the availability of I+PFC services and their provision to the System Operator (SO). However, traditional operation algorithms of distribution systems do not consider the provision of I+PFC services to the SO, though a quantification of the capability of active distribution networks to provide these services has been recently reported in [5]. Further, not only distribution networks through their Distributed Generators (DGs) can deliver I+PFC services to the SO but also Virtual Power Plants (VPPs), grouping and operating DGs in a coordinated way [6].

To this end, the objective of this work is to develop an algorithm that guarantees the evacuation of I+PFC services offered by elements connected to the distribution grid. The algorithm is formulated as an OPF. Currently, the capacity of the distribution grid is sufficient to supply the maximum demand and to evacuate the maximum amount of distributed generation. To guarantee the provision of I+PFC services to the SO, the operation of the distribution system should: (i) plan for capacity and voltage margins and (ii) re-dispatch elements providing I+PFC services (if necessary) considering the network characteristics. The Distribution System Operator (DSO) analyzes the grid limitations and re-dispatches elements by maintaining the overall offered I+PFC services to the SO at a constant value. Considering that the timescale for inertia provision is very short, whereas PFC service provision can last up to 15 minutes, the focus of this work is on PFC service. It should be noted, however, that the formulation proposed is generic enough to be utilized for provision of both services if required. Finally, the formulation can be also applied to VPP with an internal grid where a VPP operator re-dispatches its generating units for PFC service.

Following this background, the contribution of this paper is the development of an OPF-based algorithm that guarantees the provision of PFC services and that incorporates the following features:

(1) Provision of PFC service to the SO by the distribution system. Customarily, frequency constraint and thus PFC has been included in security constrained-OPFs at system level [7], [8];
(2) Implementation of a linearized AC-Power Flow (AC-PF) model to capture network capacity limits and voltage constraints. Usually, the grid has been represented by DC-PF [9];
(3) Considering full activation of the reserve through PFC service provision. Previous studies have usually consid-

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ered only the pre-disturbance power flows [10], [11].

II. Problem statement

In this work, an OPF is utilized during the operation planning of the distribution system to guarantee capacity margins and to re-dispatch elements providing I+PFC services. OPF seeks to determine the active power set points and reactive power/voltage set points to meet a given demand by optimizing a given objective function. In this case, the OPF objective function is to evacuate the offered I+PFC services to the SO at minor costs without violating operation limits.

The availability of providing I+PFC services and their quantities depends on the characteristics of these services. The Spanish operation procedures, for example, require for type C and D electricity generation modules a PFC-related capacity of 8% of the maximum capacity. RES-based electricity generation modules of type C and D can provide an inertial response of up to 10% of their capacity. For instance, the steady state power variation of element i, Δp_i^{ss}, can be computed as follows:

$$\Delta p_i^{ss} = \frac{1}{R_i}\Delta f_i^{ss} = \frac{1}{R_i}\Delta f^{ss} \tag{1}$$

where R_i is the droop and Δf^{ss} the post-disturbance steady-state frequency deviation. Thus, for a maximum steady-state frequency deviation of 200 mHz and a droop of 5%, the required power variation is 8%.

If these services were assigned through a market, the market results should be known, being an input to the OPF. With respect to the PFC, one could assume that the offered capacity corresponds to that required for post-disturbance steady state. If re-dispatch was necessary, an estimation of the available elements able to provide the service would be needed. Guaranteeing capacity margins and re-dispatching elements providing I+PFC services should consider the cost or bid functions of the elements. Indeed, re-dispatching elements leads to opportunity costs since expected benefits are modified by deviating from the power or energy schedules cleared in the markets. Since these functions are not necessarily known, market prices will be used to determine opportunity costs incurred by re-dispatching the elements. This corresponds to a weighed minimization of power and energy deviation from the market schedules.

III. Problem Formulation

This section provides the formulation of the objective function and the constraints to guarantee the evacuation of the offered I+PFC services to the SO.

A. Objective Function

The objective function weighs the deviations from the respective initial schedules (if any) according to the price:

$$f = \min\sum_k \Big[\lambda^{\mathrm{EM}}\left|p_k^{\mathrm{EM}} - P_k^{\mathrm{EM}}\right|\Delta t + \lambda^{\mathrm{PM}}\left|p_k^{\mathrm{PM}} - P_k^{\mathrm{PM}}\right| + \lambda^{\mathrm{IM}}\left|p_k^{\mathrm{IM}} - P_k^{\mathrm{IM}}\right|\Big] \tag{2}$$

where $k \in \mathcal{K}$ is the set of generating units providing I+PFC; λ^m is the price of market m (EM – energy market, PM – PFC market, IM – inertia market); P_k^m is the cleared offer of generator k in market m; and p_k^m represents the actual power delivered in real time by generator k in market m.

The objective is a nonlinear function due to the absolute differences between scheduled and actual energy/power, but a linear program is preferred as all other constraints are linear. The absolute values of the differences in (2) are replaced with: x_k, y_k and z_k, as follows:

$$f = \min\sum_k \Big[\lambda^{\mathrm{EM}} x_k \Delta t + \lambda^{\mathrm{PM}} y_k + \lambda^{\mathrm{IM}} z_k\Big] \tag{3a}$$

s.t.

$$-x_k \le p_k^{\mathrm{EM}} - P_k^{\mathrm{EM}} \le x_k\ , \quad \forall k \in \mathcal{K} \tag{3b}$$

$$-y_k \le p_k^{\mathrm{PM}} - P_k^{\mathrm{PM}} \le y_k\ , \quad \forall k \in \mathcal{K} \tag{3c}$$

$$-z_k \le p_k^{\mathrm{IM}} - P_k^{\mathrm{IM}} \le z_k\ , \quad \forall k \in \mathcal{K} \tag{3d}$$

B. Constraints

1) Total quantity of I+PFC services: The total generation should remain constant and equal to the total amount of generation cleared in the energy market, $\sum P_k^{\mathrm{EM}}$. In the same vein, the total amount of inertia and PFC services should be equal to the amount of I+PFC offers cleared in the corresponding markets (if available). In case of mandatory services, the individual contribution to PFC services for instance is at least 8%, and the total amount of PFC service would be equal to 8% of the total maximum capacity.

$$\sum_k P_k^{\mathrm{EM}} = \sum_k p_k^{\mathrm{EM}} \tag{4a}$$

$$\sum_k P_k^{\mathrm{PM}} = \sum_k p_k^{\mathrm{PM}} \tag{4b}$$

$$\sum_k P_k^{\mathrm{IM}} = \sum_k p_k^{\mathrm{IM}} \tag{4c}$$

2) Technical constraints of elements providing I+PFC services: The grid code requires the provision of at least a certain amount of I+PFC services per element. In case of market-based assignments, the market clearing determines the amount of services provided by each element. This amount can be adjusted as a function of the droop or inertia setting:

$$p_k^{\mathrm{PM}} = \frac{1}{R_k}\Delta f^{ss}\ , \qquad \forall k \in \mathcal{K} \tag{5a}$$

$$p_k^{\mathrm{IM}} = 2H_k \left.\frac{d\Delta f}{dt}\right|_{t=t_0}\ , \qquad \forall k \in \mathcal{K} \tag{5b}$$

where H_k is the inertia value and $\left.\frac{d\Delta f}{dt}\right|_{t=t_0}$ is the initial rate of change of frequency.

The active power of each element, including the possible provision of I+PFC services, as well as the reactive power (q_k) must be bounded, as follows:

$$\underline{P}_k \le p_k^{\mathrm{EM}} + p_k^{\mathrm{PM}} + p_k^{\mathrm{IM}} \le \bar{P}_k\ , \forall k \in \mathcal{K} \tag{6a}$$

$$\underline{Q}_k \le q_k \le \bar{Q}_k\ , \qquad \forall k \in \mathcal{K} \tag{6b}$$

where $\underline{P}_k, \bar{P}_k$ and $\underline{Q}_k, \bar{Q}_k$ represent the lower and upper bounds of the active and reactive power generation.

3) Power flow and voltage magnitude constraints: The active and reactive power balance at each node can be computed by using the conventional PF equations which are nonlinear. However, different linearization techniques have been proposed in the literature, depending on the involved voltage levels of the grid (HV or MV mainly) since this affects the representation of the branch impedance [12].

Here, the Logarithmic Transform Voltage Magnitude (LTVM) is used to linearize the PF equations. LTVM expresses the voltage magnitude at bus m, V_m, through a transform: $v_m = \ln|V_m|$, $\forall m \in \mathcal{B}$, where $\mathcal{B}$ is the set of buses of the power grid. It is a branch flow formulation with each line modeled as a series admittance $G_{mn} + jB_{mn}$ where G and B are the conductance and susceptance, respectively, of the line with sending end m and receiving end n. A version of LTVM with power losses on the branches and more detailed line models was reported in [13] and reproduced in (7).

$$
\begin{aligned}
p_{mn} &= p'_{mn} + 0.5 p^{\text{loss}}_{mn} , && \forall m,n \in \mathcal{B} && \text{(7a)}\\
q_{mn} &= q'_{mn} + 0.5 q^{\text{loss}}_{mn} , && \forall m,n \in \mathcal{B} && \text{(7b)}\\
p'_{mn} &= G_{mn}\left(v_m - v_n\right) - B_{mn}\left(\delta_m - \delta_n\right) , && \forall m,n \in \mathcal{B} && \text{(7c)}\\
q'_{mn} &= -B_{mn}\left(v_m - v_n\right) - G_{mn}\left(\delta_m - \delta_n\right) , && \forall m,n \in \mathcal{B} && \text{(7d)}\\
p^{\text{loss}}_{mn} &= G_{mn}\left[\left(v_m - v_n\right)^2 + \left(\delta_m - \delta_n\right)^2\right] , && \forall m,n \in \mathcal{B} && \text{(7e)}\\
q^{\text{loss}}_{mn} &= -B_{mn}\left[\left(v_m - v_n\right)^2 + \left(\delta_m - \delta_n\right)^2\right] , && \forall m,n \in \mathcal{B} && \text{(7f)}
\end{aligned}
$$

where δ_m is the voltage phase angle at bus m; p'_{mn} and q'_{mn} are the active and reactive line flows without losses respectively; and p^{loss}_{mn} and q^{loss}_{mn} are the estimated values of active and reactive power losses computed from PF runs.

For the complete optimization problem, other constraints include line flow limits (8a) and (8b), and voltage limits (8c).

$$
\begin{aligned}
-\bar{P}_{mn} \le p_{mn} &\le \bar{P}_{mn} , && \forall m,n \in \mathcal{B} && \text{(8a)}\\
-\bar{Q}_{mn} \le q_{mn} &\le \bar{Q}_{mn} , && \forall m,n \in \mathcal{B} && \text{(8b)}\\
|\bar{V}^{\min}_m| \le |v_m| &\le |\bar{V}^{\max}_m| , && \forall m \in \mathcal{B} && \text{(8c)}
\end{aligned}
$$

Finally, the nodal power balance includes the active and reactive power flows along the line.

$$
\sum_{k \in \mathcal{K}_m} p^{\text{EM}}_k = p_{mn} + p_{nm} + P^{\text{PM}}_m + \sum_{d \in \mathcal{D}_m} P_d \ , \forall m,n \in \mathcal{B} \quad \text{(9a)}
$$

$$
\sum_{k \in \mathcal{K}_m} q_k = q_{mn} + q_{nm} + \sum_{d \in \mathcal{D}_m} Q_d \ , \quad \forall m,n \in \mathcal{B} \quad \text{(9b)}
$$

where P_d and Q_d represent the active and reactive power demands; P^{PM}_m is the amount of PFC requested at bus m; and $\mathcal{K}_m$ and $\mathcal{D}_m$ are the sets of generators and demands present at bus m.

The model developed is implemented in GAMS with an interface in Excel. The user defines the parameters of the generators and the I+PFC services, the topology of the networks (buses and branches), and the loads. Flexible loads are not considered so far for re-dispatch and I+PFC provision. However, the model proposed can readily consider flexible loads, provided that upper and lower bounds on load variations are provided by the load owners, aggregators or retailers.

IV. Test Cases and Results

To ascertain the effectiveness of the model, two test cases are analyzed for the compulsory provision of I+PFC services:

(A) a simple 8-bus test system,
(B) a real Spanish HV distribution grid.

The simple test system allows to easily confirm the obtained results, whereas the application to a real distribution grid allows assessing the suitability. Since both up and downwards I+PFC services need to be considered, two scenarios are considered for the simple test case: (1) high generation, (2) high demand. These two scenarios stress the provision of upward and downward I+PFC services, respectively. Indeed, a high generation scenario poses more likely problems to additional power injection due to I+PFC service activation. Under these categorizations, each of the scenarios were further investigated under the following two broad sets.

(a) Initial dispatch without I+PFC service provision and fixed voltage magnitudes at Point of Common Coupling (PCC). In this case, the scheduled generation is checked for feasibility and presence of overloaded lines.
(b) Updated dispatch with I+PFC service provision. The model is then run with PFC provision to guarantee the provision of the scheduled services at real time.

A. 8-bus test system

Using the power network shown in Fig 1, the high generation and high demand scenarios were analysed and evaluated. The capacities of the DGs are 40 MW each whereas the RES1 is a PV of 160 MW and RES2 is an Energy Storage System (ESS) of 40 MW which can discharge and charge up to its capacity. The inertia and PFC for all generators have per capacity rates of 10% and 8%, respectively. This corresponds to a mandatory service as defined by current operation procedures.

Prices used for the energy, inertia and PFC evacuation, λ^{EM}, λ^{IM} and λ^{PM}, are €22.5/MWh, €30/MW and €25/MW, respectively in all scenarios. The considered prices are artificial and they do not necessarily reflect real prices, but they carry out a weighting function, giving here more weight to inertia

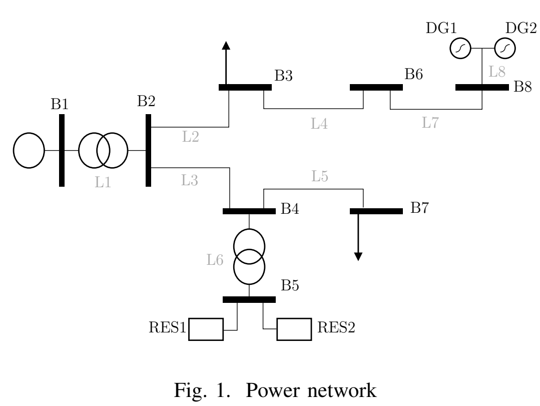

Fig. 1. Power network

than PFCs or energy. However, if a mandatory provision of I+PFCs is considered, λ^{IM} and λ^{PM} will not be used since each element has to provide the required amount of I+PFCs.

1) High Generation: The active and reactive power demands were set to 0 and all generation is transmitted to the main grid at bus B1. Table I shows the power generation schedules with the capacities and available generation of the units. The difference between generator capacity, $\bar{P}_k$, and available generation, $\check{P}_k$, is mainly of concern for RES generation that cannot increase power above the available generation.

a) Initial Dispatch without I+PFC service provision: For the initial dispatch, line flows and voltage magnitudes were within limits. The active and reactive power generation values are shown in in Table I. Figure 2 shows the active (red) and reactive (green) power flows across the network.

b) Updated dispatch with I+PFC service provision: When the model is run with PFC service provision as a constraint, some network constraints are violated (e.g., the limits of line L3). Provision of the service implies slightly additional generation from the scheduled generators and this might warrant some reschedule of some plants. In this case, it was observed that the active power generations were re-dispatched to maintain line flows within limits. The ESS at bus B5 charged less while the additional deficit was supplied by the DGs at bus B8. Active and reactive power generation values for the updated case are also shown in Table I.

In this scenario of high generation, active power was sent to the main grid through the PCC at bus B1 while reactive power was obtained from the grid. The reactive power dispatch across the network is needed to maintain the fixed voltage constraints in the network. Fig 3 shows the evolution of the active (red) and reactive (green) power flow across the network.

2) High Demand: To properly see the effect of this, DG2 was taken out of service to have a real demand-saturated case. Additionally, the PV was set to less than 10% of its maximum capacity. Active power demand at buses B3 and B7 were

TABLE I
HIGH GENERATION: INITIAL AND UPDATED DISPATCH

	$\bar{P}_k$	$\check{P}_k$	Initial		Updated	
			p_k^{EM}	q_k	p_k^{EM}	q_k
	[MW]	[MWh]	[MWh]	[MVar]	[MWh]	[MVar]
DG1	40	40	30	-15.6	33.2	23.5
DG2	40	40	30	-30	33.2	-30
PV	160	120	120	-39.9	110.4	-23.2
ESS	40	40	-20	30	-16.8	30

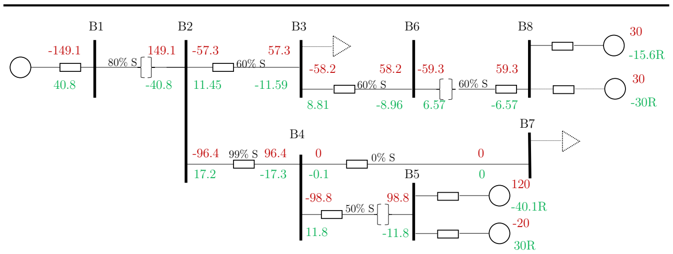

Fig. 2. High generation initial dispatch: active and reactive power flows

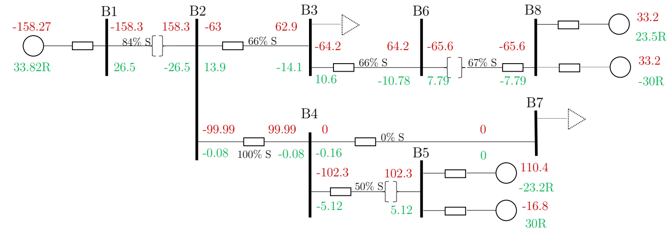

Fig. 3. High generation updated dispatch with PFC service provision: active and reactive power flows

TABLE II
HIGH DEMAND: INITIAL AND UPDATED DISPATCH

	$\bar{P}_k$	$\check{P}_k$	Initial		Updated	
			p_k^{EM}	q_k	p_k^{EM}	q_k
	[MW]	[MWh]	[MWh]	[MVar]	[MWh]	[MVar]
DG1	40	40	30	-21.6	22	8.1
PV	160	12.8	12.8	-55.9	6.8	-2.2
ESS	40	40	-12.8	30	1.2	3.2

24 and 90 MW respectively while reactive power demand at bus B3 was 18 MVar. Table II shows the power generation schedules with the capacities and available generation.

a) Initial Dispatch without I+PFC service provision: In the initial case, the provision of downwards I+PFC services would lead to an overload of line L3 (initial branch flow $>$ 95%) due to increased active power consumption.

b) Updated dispatch with I+PFC service provision: In this scenario, the ESS at bus B5 discharged instead of charging as in the initial case. Additionally, the PV and DG1 were re-dispatched to satisfy (4a). Finally, the deficit of the demands at buses B3 and B7 was supplied from the grid. Table II shows the updated active and reactive generation.

B. Spanish HV distribution grid

For the real distribution grid, there are 80 buses with 43 generators and 80 loads in service. Additionally, there are 68 lines and 17 transformers with ratings from 18 to 2286 MVA.

a) Initial Dispatch without I+PFC service provision: Although the distribution grid is interconnected with other distribution grids, the flow through these interconnections does not vary significantly when varying generation or demand within the grid. Consequently, the distribution grid is as a system with one PCC with the transmission system.

The total active and reactive power demands in the system are 194 MW and -93.7 MVar respectively. The initial offers

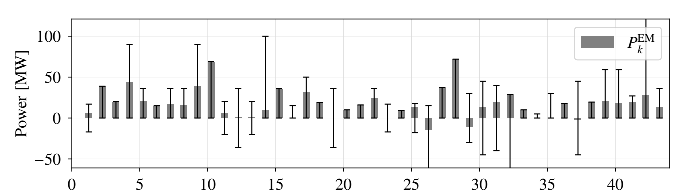

Fig. 4. HV distribution grid: Initial active power schedule and power limits

and limits of each generator are shown in Fig 4. Negative lower capacities refer to absorbing power ability. Line flows were within limits with only 3 lines above 90% capacity rating. These lines will be overloaded when PFC service is provided.

b) Updated dispatch with I+PFC service provision: When the model is run with PFC service provision requirement, solving with all the constraints led to an *infeasibility* which means some constraints were violated ((4a)). Re-dispatching generation by maintaining the overall generation and guaranteeing the feasibility of providing I+PFC service has not been possible. However, adding an unconstrained generator to the network at the PCC led to a feasible solution with the new added generator supplying the remainder of the required power. Barring this approach, (4a) was relaxed and it led to a solution but also a reduction of the overall generation. If any of the line limits was to be violated while the algorithm searches for the optimal solution, a relaxed model with less generation than expected is then utilized. The updated generation from each generator is shown in Fig 5. As observed in the figure, fifteen generators reduced their initial generation schedule; generators $k = 2$, $k = 3$, $k = 6$, etc. On the other hand, only four generators ($k = 4$, $k = 5$, $k = 9$, and $k = 19$) increased their initial output to compensate for the reduction. Generator $k = 19$, an ESS, went from charging to discharging state in order to make up for the generation in the network.

With respect to the objective function for PFC service provision, the objective value for this test case is €3384. Whereas the objective function gives an indication of the total amount of re-dispatched power, it cannot be interpreted as the value of providing I+PFC services. In this case, 150 MW was re-dispatched among the generating units in the network to comply with the network requirements. Since a compulsory I+PFC service provision is studied, the obtained objective value is here directly proportional to the amount of MW re-dispatched from the energy market schedule. In fact, when $\lambda^{\text{EM}} = 1$, the objective function coincides with the total amount of re-dispatched power.

Table III shows the lines with highest rating; it confirms that the OPF works since no line is overloaded after re-dispatching.

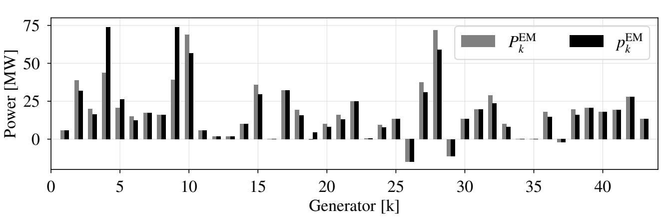

Fig. 5. Comparison of the initial and updated active power schedule of the Spanish HV distribution grid

V. Conclusions

An algorithm has been developed that guarantees the evacuation of I+PFC services offered by elements connected to the distribution grid. The algorithm is formulated as an OPF whose goal is to guarantee the evacuation of the offered I+PFC services to the SO at minor costs without violating operation limits. A major difficulty for OPF is the non-linearity of the AC PF equations and in this case, an approximated linear PF model has been set up. The algorithm has been applied to two different test cases. The first test case correspond to a very simple distribution grid and allows validating the developed algorithm for both, upwards and downwards I+PFC service provision. The second test case corresponds to a real HV distribution grid. In all cases, the algorithm has successfully re-dispatched generation to guarantee the provision of I+PFC services without violating network constraints. Future directions for this work include applying it to a system with an active I+PFC market where each generator can decide how much power to be offered in the respective market(s). Additionally, investigating network security constraints with the evacuation tool is another study direction being considered by the authors.

TABLE III
HV distribution grid test case: Branch flows

From bus	**To bus**	p_{mn} [MW]	q_{mn} [MVar]	S_{mn} [MVA]	% of Line Rating
14002	30957	-599.0331	30.7673	599.8227	99.97
30546	30604	79.4207	23.2687	82.7592	99.71
30957	31268	-110.8027	0.4249	110.803	99.91

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