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Title

Joint Energy and Capacity Equilibrium Model for Centralized and Behind-the-Meter Distributed Generation

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Abstract

This paper presents a conjectured-price-response equilibrium approach for modeling both centralized generation (CG) and behind-the-meter distributed generation (BMDG). A Nash game is set up with two constraints linking the CG and BMDG decisions to satisfy both the electricity demand in an energy market and the firm capacity in a capacity market. CG agents maximize their market profits while BMDG customers minimize their net supply costs, making decisions on their annual capacity investments and hourly productions decisions. Customers' costs account for 1) the energy bought from the grid minus the BMDG energy surpluses sold; 2) the payment of the grid access tariff (power and energy-based terms) and 3) the BMDG capacity investments' costs. The equilibrium conditions enable to represent different degrees of oligopoly using conjectural variations in both the energy and capacity markets. This work proves that such an equilibrium problem can be solved through an equivalent, yet simpler-to-solve, quadratic minimization problem. Some case examples compare the results of the proposed joint energy and capacity equilibrium with those from an energy-only equilibrium. Among other conclusions, these cases show that the proposed equilibrium sends adequate economic signals to the consumers to taper off the total system peak demand, whenever the weight of the power-based term of the access tariff is not extremely high.

Keywords

Generation expansion, distributed generation, energy and capacity markets, capacity payments, Nash equilibrium.

Nomenclature

Acronyms

CCGT	Combined Cycle Gas Turbines
CG	Centralized Generation
CV	Conjectural Variations
DG	Distributed Generation
BMDG	Behind-the-Meter Distributed Generation
EOE	Energy-Only Equilibrium
EPEC	Equilibrium Problem with Equilibrium Constraints
GA	Genetic Algorithm

GENCOs	Generation Companies
JECE	Joint Energy and Capacity Equilibrium
KKT	Karush-Kuhn-Tucker conditions
MCP	Mixed Complementary Problem
MPEC	Mathematical Problem with Equilibrium Constraints
NCP	Non-linear Complementary Problem
OCGT	Open Cycle Gas Turbines
RG	Renewable Generation
SPV	Solar Photovoltaic

Sets and indexes

C	Set of Customer segments
G	Set of GENCOs
s	Agent, $s \in C \cup G$
c	Customer segment, $c \in C$
g	GENCO, $g \in G$
t	Generation unit
h	Hour
y	Year
H^M_y	Annual peak demand hours

Parameters (uppercase and greek)

$TP_{c,y}$	Power term of the grid access tariff [€/MW]
$TV_{c,y}$	Volumetric (energy) selling term of the grid access tariff [€/MWh]
$TC_{c,y}$	Volumetric (energy) buying term of the grid access tariff [€/MWh]
$IC_{t,s,y}$	Investment costs [€/MW]
$D_{c,h,y}$	Customer's base demand [MWh]
$VC_{t,g,y}$	Production cost of CG [€/MWh]
DI_t	Firm capacity coefficient [MW/MW]
CI	Firm capacity security margin over demand peak [%]
$\theta^S_{g,h,y}$	GENCOs energy conjectures [€/MWh/MW]
$\theta_{c,h,y}$	Customers energy market conjectures [€/MWh/MW]
$\theta^L_{g,h,y}$	GENCOs capacity market conjectures [€/MW/MW]
W	Discount rate [%]

Variables (primal: lowercase, dual: greek)

$cp_{c,y}$	Customer's annual contracted power from the grid [MW]
$p_{t,s,y}$	Annual installed generation capacity (either CG or BMDG) [MW]
$dq_{c,h,y}$	Customer's hourly net demand consumed from the grid [MWh]
$eq_{c,h,y}$	Customer's hourly net injected power into the grid [MWh]
$q_{t,s,h,y}$	Hourly generated energy (either CG or BMDG) [MWh]
d^M_h	Annual net grid peak demand [MWh]
$\lambda^E_{h,y}$	Hourly energy market price [€/MWh]
λ^C_y	Annual capacity market price [€/MW]
$\lambda^C_{h,y}$	Hourly consumer capacity payment [€/MWh]

1. Introduction

1.1. *Problem description*

A significant change in the way electricity is generated, supplied, and consumed is ongoing in many countries with the massive penetration of distributed generation (DG). Specifically, DG from renewable generation (RG), like solar photovoltaic (SPV), is becoming an alternative to classic large-scale centralized generation (CG), being the Behind-the-Meter DG (BMDG, [1]), the most attractive option in this regard.

The expansion of BMDG, as it is recommended in the EU energy strategy ([2]), raises concerns on how it might affect the power sector and how conventional utilities and competitive and regulated activities may be forced to evolve. As discussed in [1], BMDG investments could reduce the overall grid system incomes due to a decline in the net grid demand (load-defection), and might trigger the so-called *death spiral*. Moreover, capacity remuneration mechanisms, in the form of capacity auctions [3], are more and more often selected as the way to ensure enough firm capacity to guarantee the system balance in a context of a large penetration of BMDG. Truly, these mechanisms can foster the investments on generation and storage capacity needed to keep the security of supply by reducing the uncertainty associated with the volatility of the RG used in the BMDG, and also limiting episodes of null and spiked prices.

This paper deals with the detailed modeling of the interactions between CG and BMDG in power systems under an operation and capacity investments decisions framework. It explicitly models both, an energy and a capacity market, and analyses the implications of the existence of the latter on the expansion of both CG and BMDG and the impact of different grid access tariff structures.

1.2. *Literature review*

Different models have been proposed in the literature to help in these kinds of analyses, representing both CG and BMDG. An up-to-date literature review was presented in [1]. However, existing models suffer from a lack of realistic approaches, mainly due to: 1) a non-hourly temporal horizon, required for an appropriate representation of the interactions among RG, storage, ramps, reserves, and startup and shutdown decisions of thermal plants; and 2) the fact that BMDG planning models do not generally consider its impact on the wholesale electricity price, which is one of the major drivers for investments.

Table 1 presents a literature summary on generation expansion models based either on minimization problems with endogenous system marginal costs or on Nash equilibria with endogenous marginal prices. For each one, the following aspects, relevant for our study, are identified: 1) the type of game considered ([4]); 2) if they have an hourly time representation; 3) if they include BMDG investments and 4) if they represent a Joint Energy and Capacity Equilibrium (JECE). Some other relevant features for each reference are also commented.

Table 1: Generation expansion models with Nash equilibria

	Game type (Cournot or conjectural)	**Hourly dispatch**	**Cost minimization or equilibrium**	**Type of resolution method**	**BMDG**	**JECE**	**Features**
[5]	Cournot	No	Equilibrium	Custom algorithm	No	No	Two-stage sequential model. Equilibrium with relaxed integrality constraints.
[6]	Cournot	No	Equilibrium	Custom algorithm	No	No	Applied to a small case study
[7]	Cournot	No	Equilibrium	MPEC	No	No	EPEC. Interesting theoretical study but not applied to a real case.
[8]	Cournot	No	Equilibrium	Optimization	No	No	Stochastic demand. Applied to the Finnish system.
[9]	Conjectural	No	Equilibrium	MPEC	No	No	MPEC (Bilevel, investment & operation). Applied to a small case study.
[10]	Conjectural	No	Cost-minimization and equilibrium	MPEC	No	No	MPEC and NCP. Stochastic demand and RES. Applied to the Danish system
[11]	Cournot	No	Equilibrium	MCP	No	No	MCP. Transmission expansion modeled. Applied to a small case study
[12]	Cournot	No	Cost-minimization	Genetic algorithm (GA)	No	No	GA. Applied to a small case study
[13]	Conjectural	No	Equilibrium	MPEC	No	No	Optimization-based resolution and MPEC (Bilevel, investment & operation).
[14]	Cournot	No	Equilibrium	NCP	No	Yes	Stochastic fuel prices, allowances, and allocations. Risk functions. NCP. Applied to a small case study.
[15]	Cournot	Yes	Equilibrium	MILP	No	No	Models auctions for a virtual power plant considering stochasticity and risk aversion.
[16]	Cournot	Yes	Cost-minimization	Custom algorithm	No	No	Energy storage applied to microgrids to improve RES performance.
[17]	Conjectural	No	Equilibrium	Custom Algorithm	No	No	Analyzes market power and collusion in double-sided power markets.
[1]	Conjectural	Yes	Equilibrium	Optimization	Yes	No	Optimization-based resolution. Applied to the Spanish system.
This paper	Conjectural	Yes	Equilibrium	Optimization	Yes	Yes	Optimization-based resolution. Applied to the Spanish system.

1.3. Main contributions

As highlighted in the last row of Table 1, this paper proposes a new hourly conjectural Nash equilibrium problem. This problem models both CG and BMDG operation and capacity expansion, with endogenous marginal prices (i.e. prices are outputs of the operation and expansion problems) and price-response conjectures [18] for both the energy and capacity markets. Moreover:

- The proposed model is built on top of the one presented in [1] but includes, as in [14], a capacity market to ensure the security of supply by satisfying a firm capacity above a predefined target margin higher than the net peak demand.
- The model endogenously computes the marginal capacity price. This price provides the signal to CG companies (GENCOs) to invest, for example, in firm peak generation capacity (highly dispatchable plants). It also allows GENCOs to recover their capacity investment costs without relying on the very high prices of power scarcity hours, as it would happen in an EOE market ([19]).

- This work also proposes an hourly mechanism for integrating the capacity payment into the hourly electricity price for the final customers.
- The model represents the secondary reserve dispatch following the ideas of [20] and [21]. However, for the sake of simplicity, reserve modeling is not formulated in this paper, although system reserve requirements are met in the case studies, as they are necessary to cover short-term RG unexpected deviations.
- As explained later, the indirect role of BMDG, which does not actively participate in the capacity market, is considered in the model. Indeed, BMDG can reduce the peak of the net grid demand finally consumed and, consequently, the system capacity needs.

Mathematically, the proposed model:

- Represents both CG and BMDG investments using a single-level equilibrium formulation for both investment and operation decisions, unlike [13] which uses a bi-level mathematical formulation.
- Can be solved with a novel equivalent quadratic optimization model derived from the KKT conditions of the JECE, as in [22].
- Includes, as in [1], operational constraints such as power gradients, start-ups and shut-downs, etc. in an hourly and detailed way.

1.4. Paper layout

This paper is structured as follows. Sections 1 and 2 formulate the proposed JECE model and the equivalent quadratic optimization model to solve the equilibrium conditions, respectively. Section 3 presents a case study that compares the results of the JECE with an EOE market, illustrating some of the main contributions of the proposed model, and finally section 4 draws the main conclusions and describes future works.

1. The CG-BMDG conjectural JECE Model

1.1. Hypotheses

This work assumes that:

- Access tariffs are inputs, and therefore the problem of determining how load/grid defections can endogenously modify these tariffs is not addressed in this paper. Indeed, the model is applied to a real size Spain-like case study for the period 2019-2039 assuming different grid access tariff structures. It compares the results of the proposed JECE with those from the EOE presented in [1], which had no explicit firm capacity requirements other than just meeting the hourly generation-demand balance.
- Each BMDG customer has an hourly electricity consumption (denoted throughout this work as the customer's base demand $D_{c,h,y}$) and the capability of installing BMDG facilities (at a given investment cost) to self-consume and sell the surplus to the grid.
- BMDG is SPV with storage back-up and zero production costs.
- The following schema of incomes and payments of the consumer from/to the grid is considered (simpler modeling could be considered by nulling some of these items):
  1. Purchase and sell-of-surpluses of electricity from/to the grid, valued at the energy market price, based on an hourly energy net metering.

2. Payment of a regulated grid access tariff in the form of 1) a power-related term (applied to the power contracted by the customer) and/or 2) an energy-related term (a volumetric term applied to the net energy exchanged with the grid), which can be different when importing or exporting from/to the grid. The annual power contracted with the grid is assumed to be the annual peak net demand of the customer.
3. Whenever the consumer buys energy from the grid, it is assumed that it also purchases generation capacity valued at the capacity market price. Whenever the net exchange with the grid is positive (the consumer exports to the grid) it is also reimbursed (being an income) for its contribution to reduce the net peak demand of the system.

1.2. The JECE formulation

Following the scheme adopted in [1], two different optimization problems are linked together to build the proposed JECE model. The first one addresses the simultaneous maximization of the GENCOs' profits. The second one addresses the simultaneous minimization of the expenses of BMDG customers. Investment costs in new generation capacity are considered in both types of optimizations. The model represents the individual decision-making rationality of each one of the market agents (both GENCOS and BMDG customers), taking also into account the reaction of the competitors by employing price-response conjectures, leading altogether to a joint energy and capacity conjectural Nash equilibrium.

Next, the objective functions and main constraints of the proposed model are described in detail. The detailed hourly operation and investment constraints for CG and BMDG are those described in [1] which, for the sake of clarity, are not repeated here. Only the two linking constraints that set the game, that is, the generation and demand balance constraint in the energy market, and the maximum firm capacity constraint, are described in this paper. In addition, the generation and demand balance constraint for each BMDG customer is also described.

1.3. The game-setup constraints

1.3.1. The BMDG customer generation and demand balance constraint.

For each customer segment c, its hourly base demand $D_{c,h,y}$ is met by the energy self-produced $q_{t,c,h,y}$ and/or bought from the grid $dq_{c,h,y}$. If the customer self-produces more than its base demand, the surplus $eq_{c,h,y}$ is sold back to the grid. Both conditions are represented in equation (1).

$$\sum_{t} q_{t,c,h,y} + dq_{c,h,y} = D_{c,h,y} + eq_{c,h,y} \qquad \forall c; \forall h;\ \forall y \tag{1}$$

where both $dq_{c,h,y}$ and $eq_{c,h,y}$ can never be simultaneously positive (since a customer cannot buy from and simultaneously inject energy to the grid, as the customer would have to pay twice for the access tariffs of that energy).

1.3.2. The wholesale generation and demand balance constraint.

The total customers net demand, $\Sigma_c dq_{c,h,y}$ - $\Sigma_c eq_{c,h,y}$, is met with the total CG production in the energy market:

$$\sum_{t,g} q_{t,g,h,y} = \sum_{c} dq_{c,h,y} - \sum_{c} eq_{c,h,y} : \lambda^{E}_{h,y} \qquad \forall h;\ \forall y \tag{2}$$

Constraint (2) links CG and BMDG productions and therefore sets the energy market game. It constraint does not include the non-supplied energy. The energy market price is set to its dual variable $\lambda^{E}_{h,y}$. By simplicity, transmission and distribution network losses are ignored, though they could be considered in a simplified manner as explained in [1].

1.3.3. The capacity market balance constraint.

The firm capacity required in the system is set annually as a margin (1+CI) over the annual grid peak demand. The latter corresponds to the maximum value, over a full year, of the hourly energy extracted from the grid by all customers together, that is $\max_h(\sum_c dq_{c,h,y} - \sum_c eq_{c,h,y})$. This firm-capacity must be met by the total firm capacity of the installed generation of the system. In this sense, each CG contributes to the firm capacity in proportion to its installed capacity $p_{t,g,y}$ times its firm capacity factor DI_t, which is generation technology *t* dependent:

$$\sum_{t,g} DI_t \cdot p_{t,g,y} = (1 + CI) \cdot \max_h \left(\sum_c dq_{c,h,y} - \sum_c eq_{c,h,y} \right) : \lambda_y^C \qquad \forall y \qquad (3)$$

The firm capacity factor DI_t represents the CG capacity expected to be available whenever needed (typically in peak hours) with a probability larger than a pre-set value (for instance 90%). Therefore, RG firm capacity factor DI_t is typically much lower than that of conventional generators, see for example Table 2 in section IV.

Note that:

- Constraint (3) links the CG power installed capacities and the DG productions, setting the game that represents the firm capacity market. The capacity market price is set to its dual variable λ^{C}_{y}.
- Price λ^{C}_{y} does not depend on *h* since the capacity market is run on an annual basis[1]. International energy exchanges are not considered in (3), which guarantees a certain level of national self-sufficiency.
- At each hour, the consumers' surplus of energy sold to the grid $eq_{c,h,y}$ can meet other clients' net demand $dq_{c,h,y}$. As a consequence, these surpluses contribute to reducing the net peak demand of the electricity system (modeled through the maximum in (3)).
- Note that 1 MW of installed capacity of BMDG reduces the mentioned net peak demand according to its corresponding generation at the net peak hour (which in the case of SPV depends on the existing solar radiation). However, CG is required to cover the peak reduced by the firmness factor DI_t. Since this factor is based on worst-case scenarios (which is the usual assumption in reliability analyses), the same worst-case strategy to represent BMDG situations should also be used for the analysis of the system security.

An auxiliary variable d^{M}_{y} to model the grid net demand peak is used to avoid the non-differentiable maximum operator in the right-hand side of (3), so that (3) is rewritten as:

$$\sum_{t,g} DI_t \cdot p_{t,g,y} \geq (1 + CI) \cdot d_y^M : \lambda_y^C \qquad \forall y \qquad (4)$$

[1] The capacity market might be designed to cover several years ahead together. The extension of this in the modeling is straightforward, by including explicit mathematical summations. This can be useful when modeling real capacity auctions that typically consider bids for three- or four-year horizons.

$$\sum_{c} dq_{c,h,y} - \sum_{c} eq_{c,h,y} \leq d_y^M : \lambda_{h,y}^C \qquad \forall h;\ \forall y \tag{5}$$

Note that in (5), $d^M{}_y$ will automatically fit the annual maximum hourly grid demand (the larger value of the left-hand side for all hours) since increasing $d^M{}_y$ over that value will not be profitable. Indeed, this would imply, according to (4), higher investments in power capacity that are finally not used. Therefore, if $H^M{}_y$ denotes the non-empty set of peak hours in year y, the dual variables $\lambda^C{}_{h,y}$ of (5) are all null except for the hours in $H^M{}_y$ (since those hours are the only ones that bind (5)).

1.4. CG objective functions

Each GENCO g maximizes the net present value of its profits, that is, the incomes from both the energy and capacity markets minus the investment and the operation costs:

$$\text{Max}\left\{\sum_{y}(1+W)^{-y} \cdot \left[\sum_{t,h}\left(\left(\lambda_{h,y}^{E} - VC_{t,g,y}\right) \cdot q_{t,g,h,y}\right) + \sum_{t}\left(\left(\lambda_{y}^{C} \cdot DI_{t} - IC_{t,g,y}\right) \cdot p_{t,g,y}\right)\right]\right\}, \forall g \tag{6}$$

The first term represents the energy market incomes (hourly energy market prices $\lambda_{h,y}^{E}$ times the hourly production $q_{t,g,h,y}$) minus the operation costs. The second term represents the capacity market incomes (annual firm capacity price $\lambda^C{}_y$ times the firm capacity $DI_t \cdot p_{t,g,y}$, being this term the main change with respect to the GENCOs' objective function already in place in [1]), minus the investment costs ($IC_{t,g,y}$ times the installed capacity $p_{t,g,y}$). Note that it is the amount of firm capacity, and not the installed capacity, that is remunerated in the capacity market.

1.5. BMDG objective functions

Each BMDG customer segment c minimizes the net present value of its net expenses, that is, its costs minus its incomes. The costs encompass:

1) The BMDG investment costs (first summatory inside the bracket [] in (7)).
2) The cost of purchasing the energy from the grid at the energy market price $\lambda_{h,y}^{E}$ and the cost of purchasing the firm capacity at the capacity price $\lambda^C{}_{h,y}$ (second summatory inside the bracket [] in (7)).
3) The incomes correspond to the sales of surpluses in the energy market at the energy spot price $\lambda^E{}_{h,y}$ and the remuneration associated with its contribution to reducing the net peak demand of the system, valued at the hourly capacity price $\lambda^C{}_{h,y}$ (third summatory inside the bracket [] in (7)).
4) The cost of paying the power-based term $TP_{c,y}$ when contracting the power $cp_{c,y}$ (the term without a summatory inside the bracket [] in (7)).
5) The volumetric term which in turns may be different for purchases $TV_{c,y}$ or sales $TC_{c,y}$ from/to the grid (the fourth summatory inside the bracket [] in (7)).

$$\text{Min}\left\{\sum_y (1+W)^{-y} \cdot \left[\sum_t IC_{t,c,y} \cdot p_{t,c,y} + \sum_h \left(\lambda^E_{h,y} + \lambda^C_{h,y}\right) \cdot dq_{c,h,y} - \sum_h \left(\lambda^E_{h,y} + \lambda^C_{h,y}\right) \cdot eq_{c,h,y} + TP_{c,y} \cdot cp_{c,y} + \sum_h \left(TV_{c,y} \cdot dq_{c,h,y} + TC_{c,y} \cdot eq_{c,h,y}\right)\right]\right\}, \forall c \tag{7}$$

Regarding the formulation previously presented in [1], (7) adds the hourly capacity prices $\lambda^C_{h,y}$ for customers to pay[2] for the energy required from the grid during peak hours H^M_y. Prices $\lambda^C_{h,y}$ must be set to guarantee that the total net cost paid by the customers equals the total incomes that GENCOs must receive from the capacity market, i.e.:

$$\sum_{c,h,y} \left(dq_{c,h,y} - eq_{c,h,y}\right) \cdot \lambda^C_{h,y} = \sum_{t,g,y} \lambda^C_y \cdot DI_t \cdot p_{t,g,y} \tag{8}$$

The annex proves that (8) holds true when $\lambda^C_{h,y}$ are the dual variables of (5). Finally, the customer contracted power, for which the customer will be charged with the power term of the access-tariff, is the maximum power ever demanded from or sold to (not simultaneously) the grid:

$$dq_{c,h,y} + eq_{c,h,y} \leq cp_{c,y} \qquad \forall y \tag{9}$$

1.6. Final game formulation

The final game is defined through the simultaneous optimization, in the sense of Nash, of (6) and (7), subject to (10) (for $s \in G$) and to (1) and (10) (for $s \in C$), being (2), (4) and (5) the linking constraints of the game. Finally, constraint (10) guarantees that the production $q_{t,s,h,y}$ for either BMDG or CG never exceeds the installed capacity:

$$q_{t,s,h,y} \leq p_{t,s,y}, s \in C \cup G \tag{10}$$

2. The equivalent Cost Minimization Model

A conjectural variations (CV) approach [4] is applied to the game described above, where the same price-response conjectures as in [1] are used for the energy market (see [1] for an explanation of the inequalities shown in (11)):

$$\frac{\partial \lambda^E_{h,y}}{\partial\left(\sum_t q_{t,g,h,y}\right)} = -\theta^E_{g,h,y} \tag{11}$$

[2] Consider that, if during these hours $eq_{c,h,y}$ is positive (i.e. if a customer segment is selling energy), they will receive remuneration for this energy, as they are helping to reduce the system peak demand. However, as this would only be active during the peak demand hours, it is highly unlikely to happen.

$$\frac{\partial\left(\lambda_{h,y}^{E}+\lambda_{h,y}^{C}\right)}{\partial dq_{c,h,y}}=-\theta_{c,h,y}=-\frac{\partial\left(\lambda_{h,y}^{E}+\lambda_{h,y}^{C}\right)}{\partial eq_{c,h,y}} \tag{12}$$

Similarly, the conjectures for the capacity market can be defined as the residual demand slope [23] of each GENCO in said market, i.e. (see [9] for the equivalent conjecture for the energy market):

$$\frac{\partial\lambda_{y}^{C}}{\partial\left(\sum_{t}p_{t,g,y}\right)}=-\theta_{g,y}^{C} \tag{13}$$

Crossed conjectures between the energy and capacity markets (for example, the derivative of the electricity price $\lambda^{E}_{h,y}$ with respect to the capacities $p_{t,s,y}$) can be considered null (as in [24] for the energy and reserve markets). This is because the residual demand in each market only depends on the specific commodity traded in that market (energy in the energy market and capacity in the capacity market) since separated merit-order based dispatch is assumed (see [20] for the definition of other types of dispatches, like for example the so-called joint dispatches).

Assuming (11), (13) and null crossed conjectures, the proposed equilibrium model can be transformed into the following equivalent quadratic minimization problem, as it is proved in the annex:

$$\operatorname{Min}\left\{\begin{array}{l}\displaystyle\sum_{g}\left(\begin{array}{l}\displaystyle\sum_{h,y}\left(\begin{array}{l}\displaystyle\sum_{t}VC_{t,g,y}\cdot q_{t,g,h,y}+\\ \displaystyle\theta_{g,h,y}^{E}\cdot 0.5\cdot\left(\sum_{t}q_{t,g,h,y}\right)^{2}\end{array}\right)\\ \displaystyle+\sum_{y}\left(\begin{array}{l}\displaystyle\sum_{t}IC_{t,g,y}\cdot p_{t,g,y}+\\ \displaystyle\theta_{g,y}^{C}\cdot 0{,}5\cdot\left(\sum_{t}DI_{t}\cdot p_{t,g,h,y}\right)^{2}\end{array}\right)\end{array}\right)\\ \displaystyle+\sum_{c}\left(\begin{array}{l}\displaystyle\sum_{y}TP_{c,y}\cdot cp_{c,y}+\sum_{t,y}IC_{t,c,y}\cdot p_{t,c,y}\\ \displaystyle+\sum_{h,y}\begin{bmatrix}TV_{c,y}\cdot dq_{c,h,y}+TC_{c,y}\cdot eq_{c,h,y}\\ \theta_{c,h,y}\cdot 0.5\cdot\left(eq_{c,h,y}-dq_{c,h,y}\right)^{2}\end{bmatrix}\end{array}\right)\end{array}\right\} \tag{14}$$

subject to (1), (2), (4), (5), and (10). Indeed, the annex proves that the Karush-Kuhn-Tucker (KKT) conditions of (14) coincide with the KKT conditions of the game. Note that different values for the conjectures represent different degrees of oligopoly, with null conjectures corresponding to perfect competition (since a system cost minimization problem is obtained), [18]. Note that the (14) is also a generalization of the problem in [1], which is recovered by setting null the conjectures for the capacity market (i.e. $\theta_{g,y}^{L}$ = 0) and deactivating constraint (4) (i.e. CI = -1). The computation of these conjectures is not addressed in this paper, as it is a topic for a whole different work [25].

3. Case studies and results

The case study selected to illustrate the use of the model corresponds to a real-sized Spain-like system and is similar to the case-study in [1]. The simulation period has been set to 21 years (from 2019 to 2039), using a single representative week of 168 hours per year to reduce the computational burden, according to the methodology described in [26]. This representative week can account for both the best- and the worst-case scenarios in terms of RG from BMDG. The objective is to analyze the behavior of the different agents modeled and to compare the results yielded by the joint energy and capacity equilibrium (JECE) presented in this work with the EOE presented in [1]. Input data and scenarios' descriptions are explained in sub-section 3.1 while the results and their analysis are shown from sub-section 3.2.1 on. The results are the BMDG and CG investment decisions, the energy and capacity market prices, the power contracted by the consumers, and the hourly distribution of the demand for the 24 scenarios considered. The data and results shown and analyzed hereafter focus mainly on the last simulation year (2039) since it is the most representative year (it embraces all investments decided by the model).

The model has been programmed with GAMS v28.2 and solved using CPLEX [27], on an Intel i7-4790 CPU at 3,6 GHz with 32 GB of RAM. Each case study consists of approximately 5.0 million constraints and 4.0 million variables, of which 1.9 million are discrete (such as those for start-ups and shutdowns modeling) and 2.1 million are continuous. However, in order to carry out the differentiations performed in Annex I, discrete variables were relaxed. The problem resolution lasts around 21 minutes.

3.1. Input data and scenarios' description

3.1.1. Demand profiles

The hourly demand profiles of the 12 customer segments have been extracted from [28] and [29], and are represented in Figure 1**¡Error! No se encuentra el origen de la referencia.**. Demand growth has been set to be 0.5% per year, resulting, in the last year of the simulation horizon, 2039, a total demand of 298 TWh, and an hourly peak demand of 47.4 GWh.

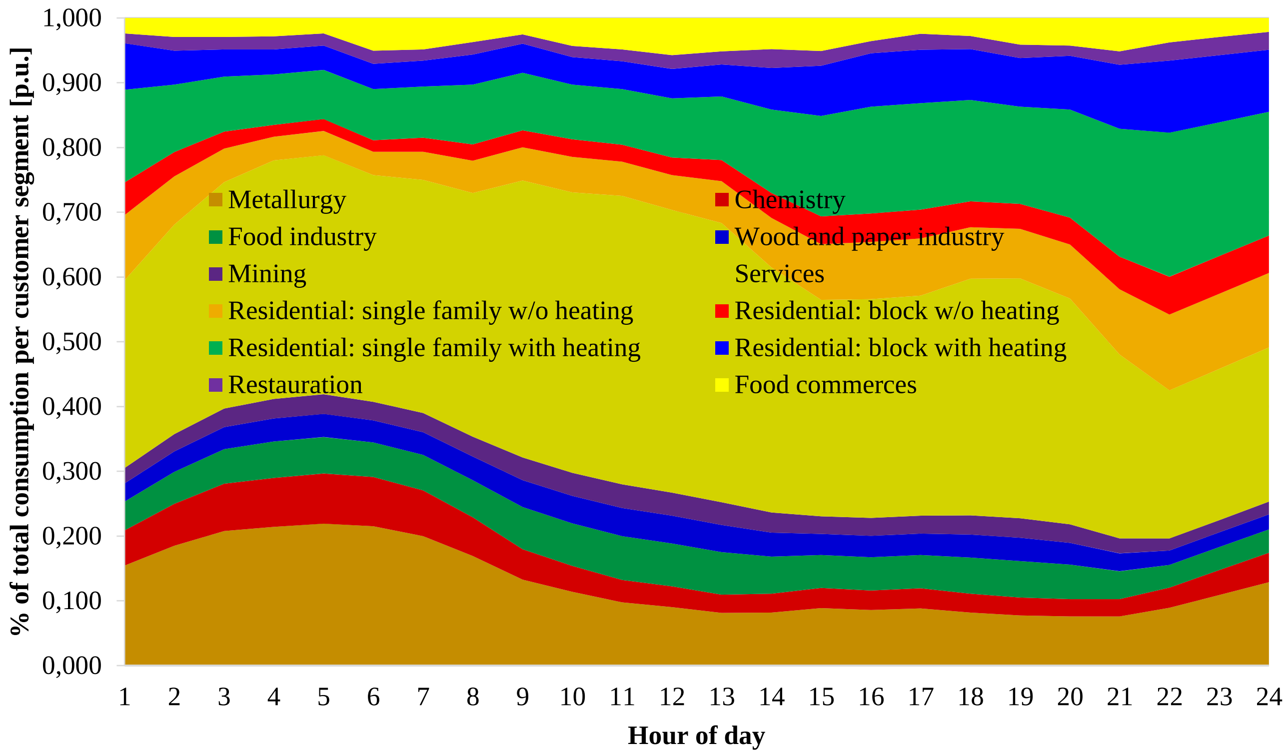

Figure 1: % of total consumption per customer segment.

3.1.2. Technologies and investment decisions

The technologies to invest in are combined-cycle gas turbines (CCGT), open-cycle gas turbines (OCGT), wind, solar photovoltaic (PV), and storage facilities (batteries) for CG, and PV and battery facilities for BMDG. Each thermal generation plant is modeled individually, including their start-up, shut-down, and ramping constraints. The expected lifespan is assumed to be 30 years for both OCGT and CCGT units, 45 years for coal-fired units, and 60 years for the currently existing nuclear power plants, so that their expected closure is considered by the model. No closure of currently existing hydro plants is considered.

Investment costs for all CG and BMDG technologies have been estimated from [30], and are considered equal across all scenarios. The time evolution of these costs (Figure 2, in real costs[3]) is set with the same methodology as in [1], and it is considered to be the same across all scenarios.

[3] All costs and prices are presented in real values. The model, however, minimizes the net present value of the costs using a discount rate of 6%.

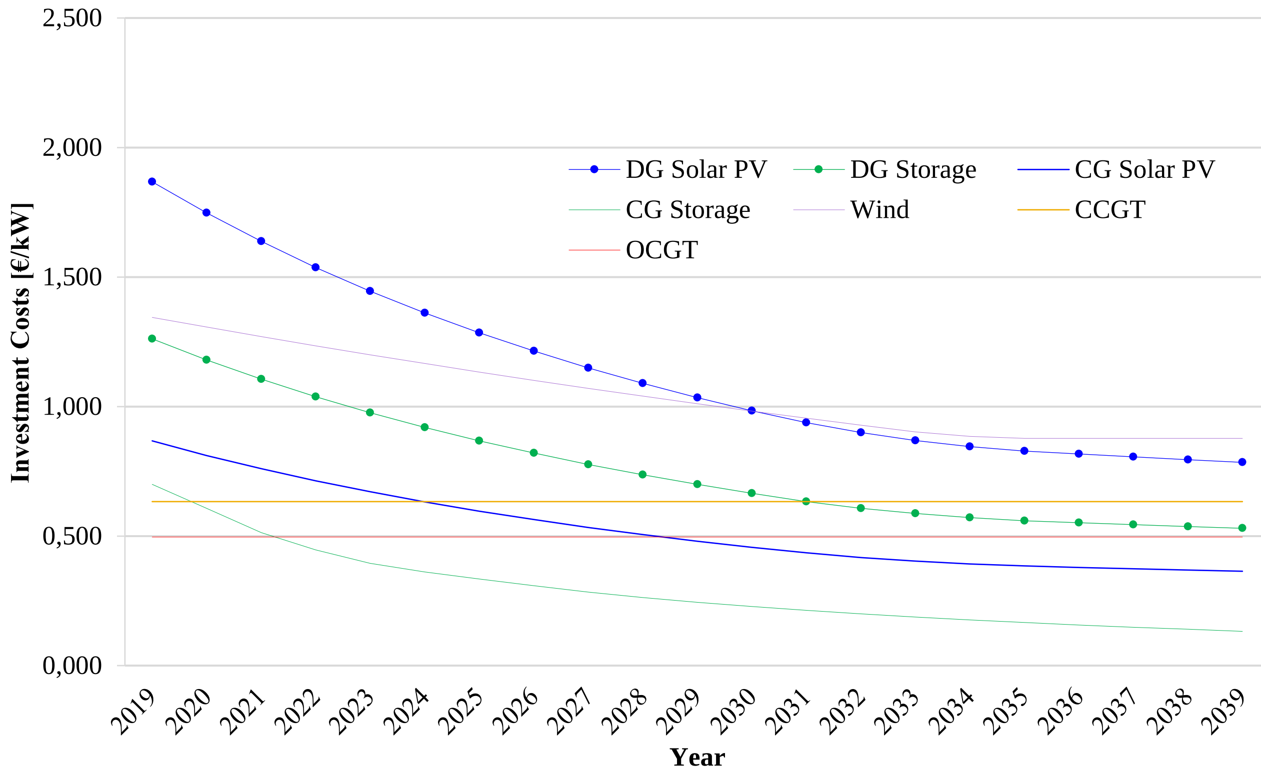

Figure 2: Generation technologies investment costs time evolution.

As can be seen, a steady decrement accounts for technological advancements in less mature technologies. However, this decrement slows down as some technologies become mature. More mature technologies' costs such as CCGT or OCGT remain constant. For the sake of clarity, regarding BMDG technologies, only the costs for industrial consumers are displayed. Storage investment costs refer to those of a 2-hour battery (i.e. the investment cost shown for storage corresponds to the installation of a 1 kW facility able to store up to 2 kWh of energy).

In the absence of more specific available information, the firmness factor (see Table 2**¡Error! No se encuentra el origen de la referencia.**) for each technology (DI_t in the constraint (4) of the JECE model) has been set equal to the availability factors of the Spanish regulator published in [31]**¡Error! No se encuentra el origen de la referencia.**, with two considerations: a) OCGT firmness factor, missing from [31], has been set to the same value (93%) as for CCGT, and b) the firmness factor of battery storage facilities has been set to 0%, assuming them to be short-term storage.

Table 2: Firmness factors for all technologies

Technology	Firmness factor DI_t [%]
Nuclear	87
Anthracite coal	90
Lignite coal	89
Other (foreign) coals	94
CCGT	93
Hydro	59
Wind	22
PV	11
OCGT[4]	93
Other	30
Battery storage	0

The values in Table 2 are the only ones publicly available from official Spanish sources related to the technologies' firmness, although they were estimated for the computation of payments of ancillary services. It can be argued that the firmness of RES generation in this table should be lower, which could be true in a system without large-scale battery storage. However, the combined operation of RES generation and battery storage in the long-term can provide firm partially dispatchable capacity, contributing significantly to meet the constantly varying demand for electricity, as well as the need for operating reserves to achieve reliable service, [32].

In general, the above firmness values should be estimated based on probabilistic studies that account for the expected firmness of each technology when energy generation is scarce, which usually occurs during demand peak hours (see for example [33] for a study for the ERCOT system).[5]

3.1.3. Conjectures

For both cases, GENCOs' and customers' energy CVs ($\theta^{S}_{g,h,y}$ and $\theta_{c,h,y}$) have been set to the same values as in [1] (estimated from historical data, [25]), and for the capacity market case, GENCOs' capacity CVs ($\theta^{L}_{g,h,y}$) are proportional to their energy CVs ($\theta^{S}_{g,h,y}$)[6]. These values can be found in Table 3**¡Error! No se encuentra el origen de la referencia.**. Note that, in the proposed model, customer segments do not have a capacity CV parameter.

[4] Although absent in [31], it was set to the same value as CCGT.

[5] Measuring how much each technology contributes to the guarantee the electricity supply is a more complex topic than considering unique firmness factors, as it depends not only on the characteristics of the technology but also on the characteristics of the whole system (the mix of generation technologies, the shape of the demand curve, etc.).

[6] Being the proportionality factor the relation between the investment and variable costs of the expected marginal technology in the capacity market (OCGT). The reason for this is that any GENCO should have the same market power in both markets of the power system.

Table 3: Energy and capacity conjectural variations for all agents.

Agent	Energy CV [€/MWh²]	Capacity CV [k€/MW²]
Large GENCOs	5	38.8
Mid-sized GENCOs	3	25.2
Small-sized GENCOs	0	0
Metallurgy and food industries	5	-
Chemistry, mining and paper industries	4	-
Food and services commerce	1	-
Other customer segments	0	-

3.1.4. Scenarios description

To analyze the impact that access tariffs and the market design (either EOE or JECE auctions) have on the evolution of CG and DG investment decisions, a set of 24 scenarios have been built and simulated. These scenarios result from 12 different combinations of the grid access tariffs and the 2 market designs, as detailed below.

Regarding the grid access tariffs, both the variable ($TV_{c,y}$ and $TC_{c,y}$) and fixed ($TP_{c,y}$) tariff terms are varied from a 100% variable tariff term weight to a 100% fixed tariff term weight, in 10% steps (11 different scenarios). In all cases, both terms have been estimated so that the overall tariff incomes allow to recover, in 2019[7], the total amount of system regulated costs as published in [34], assuming that no DG is installed in the system at that year. The current tariffs in Spain (33% of incomes coming from the variable term and 67% from the fixed term, as stated in [34]) have also been considered, so, in the end, there is a total of 12 tariff scenarios. Besides, in all cases, both variable terms are considered equal, i.e., $TV_{c,y} = TC_{c,y}$. Yearly tariff increments have not been considered.

As mentioned, regarding the market design, 2 possibilities are considered: either there is no capacity market, and the model represents an EOE market, as in [1], with constraint (4) disabled, or there is a firm capacity market in the sense of the proposed JECE by including constraint (4) with a capacity index of 10% (CI=0.1 in (4)).

Regarding the representation of uncertainty, the methodology described in [26] has been applied. This methodology builds a shorter representative period that approximates historical duration curves of the most relevant variables such as the RG and their ramps, the demand, etc. Therefore, this methodology can synthesize several years of historical data into a single week that represents, to some extent, possible scenarios in the past on both the demand and the interruptible generation, and yielding similar results to those obtained by modeling all the hours of each year (see [26] for more details). Moreover, the synthesized week approach of [26] requires less computational resources in comparison with other alternatives like for example the application of stochastic programming.

3.2. Simulation results

3.2.1. DG investments results

[7] As in 2019 there are no BMDG investments allowed, demand is constant and tariff incomes depend only on the different terms of the tariff. Therefore, these terms can be adjusted so a certain percentage of the incomes comes from the variable term, and the rest comes from the fixed term.

Figure 3 shows the cumulative DG capacity investments as a function of the weight (in %) of incomes collected by the power-based term of the access tariff over the total incomes collected by the full access tariff ("weight of the power-based tariff term" from now on, to simplify the text). Marker types in this figure refer to EOE or JECE market designs, while green and blue lines correspond, respectively, to PV and battery storage facilities:

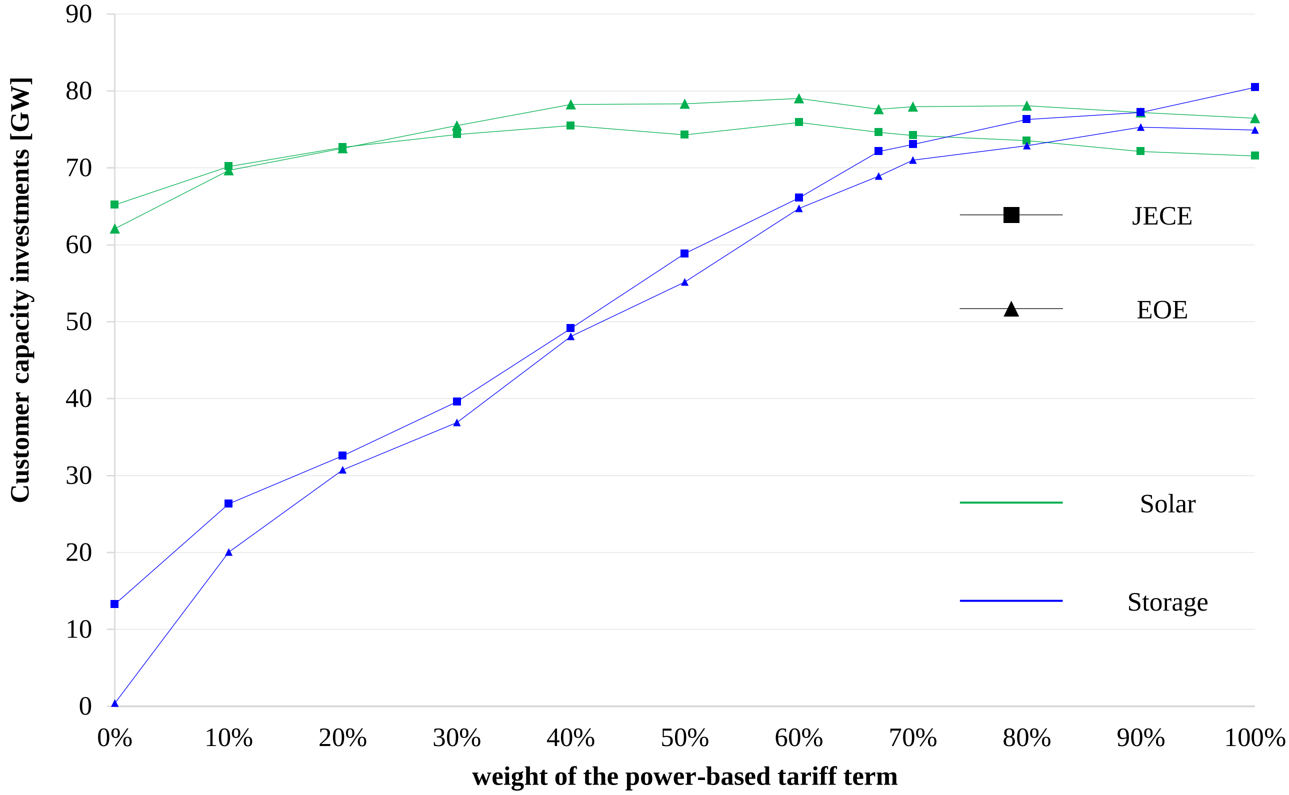

Figure 3: Cumulative BMDG investments in 2039 per weight of the power-based tariff term.

Analysis of Figure 3 results leads to the following outcomes:

- There is a positive correlation between BMDG storage investments and the weight of power-based tariff incomes, for both market designs. This could be expected, since increasing the weight of the power-based tariff term incentivizes consumers to decrease their peak demand retrieved from the grid by installing BMDG storage (storage has load-shifting capabilities and is an adequate technology for peak shaving). However, this correlation decreases for a weight of incomes of the power-based tariff term above 70%, due to a diminishing returns effect, see [35] and [36].
- DG storage investments are higher in the JECE model than in the EOE one. Indeed, in the EOE model, storage is used to reduce the power-based tariff term payments corresponding, for each customer segment, to a reduction of its individual peak demand. However, in the JECE model consumers are also interested in lowering the system peak to decrease their payments in the capacity market. This is more evident when the power-based tariff is null, since for the EOE model (with 0% of incomes of the power-based tariff term) DG storage investments are null, while for the JECE model they are not.
- DG PV investments are affected by two effects:
    - On one hand, synergies with DG storage (see [37]) cause BMDG PV to increase, as can be seen for 0% to 40% of incomes of the power-based tariff term. However, unlike BMDG storage, BMDG PV cannot be used for peak shaving, as the peak demand takes place during

nighttime, about 21:00 (according to data from the Spanish transmission system operator, [38]), which makes BMDG PV investments less profitable for higher power-based tariff terms.

- On the other hand, the energy base term of the tariff behaves as an incentive to reduce consumption from the grid by installing BMDG PV. Logically this effect is more remarkable for larger energy base tariff terms (for 0% to 40% of incomes of the power-based tariff term), as can be seen in Figure 3.
- The combination of both effects results in the inverted bathtub curves shown in Figure 3. However, the second effect is stronger in the EOE model due to higher average energy prices, causing both lines to cross for a weight close to 20%.

The following subsections show how DG storage investments are related to the reduction in the contracted power of consumers and how DG PV investments are linked with the reduction in the consumers' total demand.

3.2.2. Total customer demand and contracted power

Figure 4 shows the total net energy demanded by customers from the grid (scale on the right-hand side of the figure) and the total contracted power (CP) by customers (scale on the left-hand side of the figure) per weight of incomes collected by the power-based term of the access tariff for both market designs.

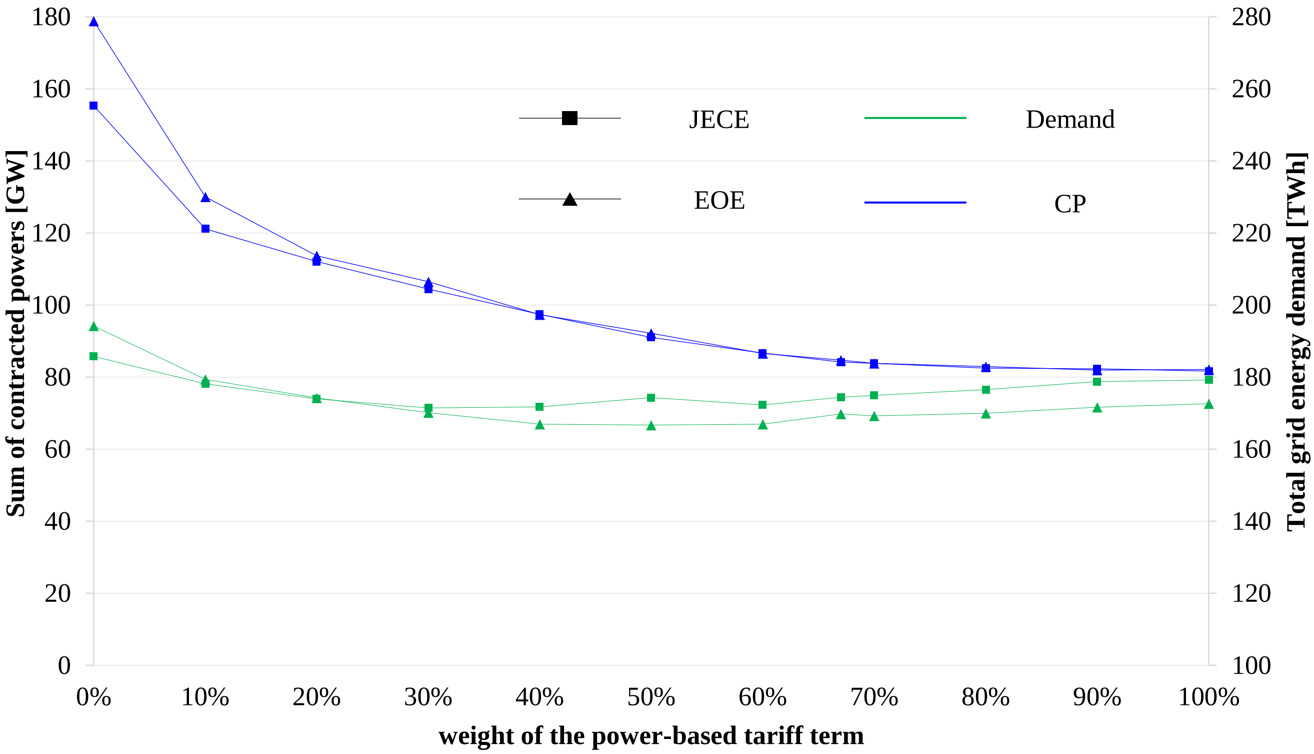

Figure 4: Total contracted power and customer demand in 2039 per weight of the power-based tariff term.

Figure 4 shows that:

- As expected, both CP curves follow an inverse path to the one followed by the DG storage investment curves shown in Figure 3. This reinforces the conclusion that one of the main drivers for installing BMDG storage is the reduction of the weight of the power-based term in the access tariff.
- There is almost no difference in the total resulting contracted power for the two different market designs whenever the weight of the power-based tariff term is above 20%. This suggests that the increment in BMDG storage investments associated with the JECE (versus the EOE) market design, identified previously in Figure 3, is not directly related to the weight of the power-based tariff term but rather to the reduction of the capacity payments that installing BMDG storage entails in the JECE (see Figure 6) which is in turn not considered in the EOE.
- Total grid demand curves show an inverse shape of that of the BMDG PV investment ones in Figure 3 (even, the lines associated to both market models cross for a weight close to 20% in both cases), indicating that load-defection (i.e., the reduction in grid consumption using self-generation to lower the energy base related tariff costs) is the main driver behind BMDG PV investments in both market models.

The following subsections analyze how these results impact the wholesale energy market by looking at the CG investments and market prices.

3.2.3. CG investments

Figure 5 shows the total CG new capacity investments (over the current existing capacity in the Spanish system) for the different weights of the power-based term of the tariff and the two market model designs.

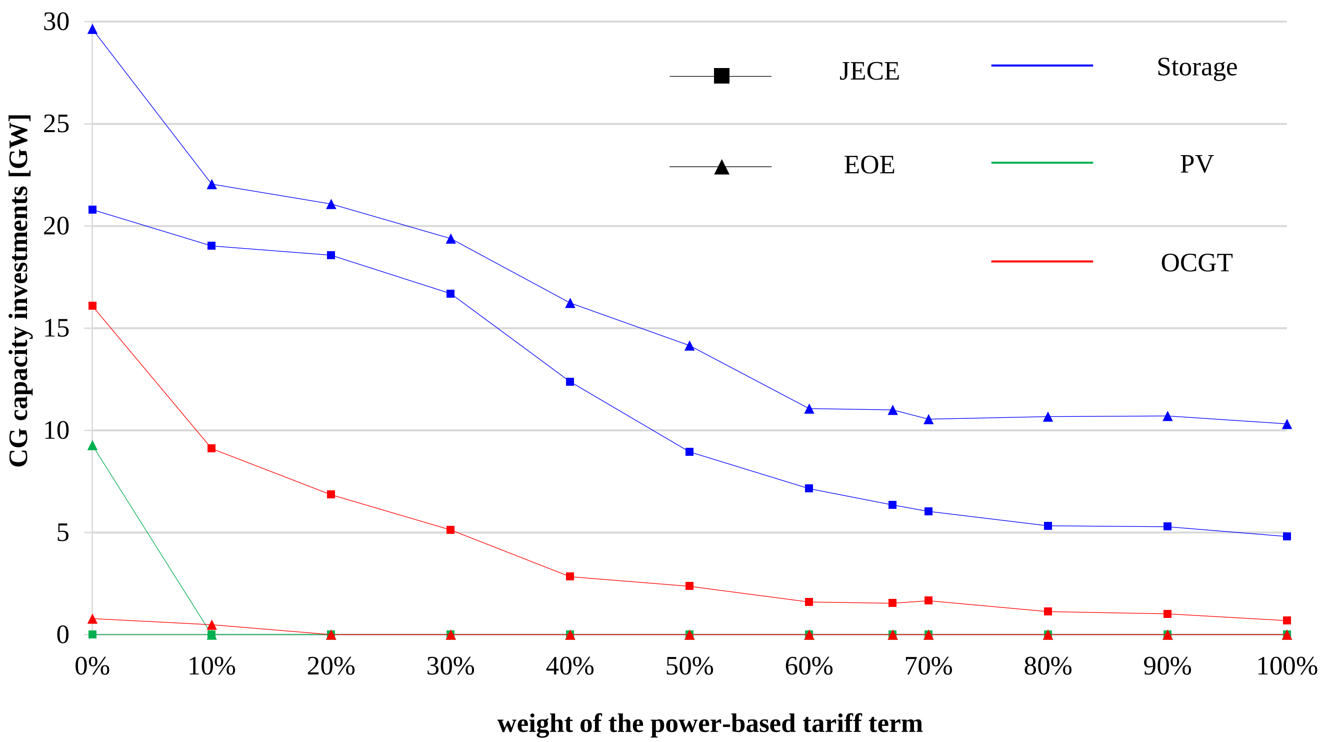

Figure 5: Cumulative CG investments in 2039 per weight of the power-based tariff term.

Although investment options consider CCGT, OCGT, wind power, PV, and storage technologies, results show that, for the set of investment costs assumed in this work (Figure 2), only new investments in OCGT, PV, and storage technologies take place. Figure 5 also shows that:

- OCGT investments take place almost only in the JECE market model option. Indeed, as this technology has both a low investment cost (lower than CCGT, wind, and PV) and a high firmness factor ($DI_{'OCGT'}$=93%), it is the perfect technology to provide the firm capacity required by constraint (4). Instead, for the EOE market the system prefers to resort to storage investments, which are not any longer fully valid for the JECE market model option, since their contribution to the firmness requirements is considered to be null in this study.
- In addition, OCGT investments in the JECE model option decrease when the weight of the power-based tariff term increases. This can be explained by the fact that, as previously commented, higher weights of the power-based tariff term incentivize consumers peak reductions (see Figure 8), thus reducing the right-hand side of constraint (4), and therefore the need for investments in CG peaker (i.e. low investment cost although high production cost) technologies.
- CG storage investments in both market model options decrease for higher weights of the power-based tariff term. This is reasonable since higher weights of the power-based tariff term incentivize flattering the demand by increasing BMDG storage investments (Figure 3), and therefore reducing the need for CG storage (Figure 8).
- As there is no firm capacity constraint in the EOE model other than the generation-demand balance, constraint (2), there is no hard requirement for OCGT investments. This leads to larger BMDG storage investments in the EOE model instead of the OCGT investments of the JECE one. CG storage investments are always significantly higher in the EOE model than in the JECE one. Moreover, it was verified that the few hours with OCGT production in the JECE model correspond to hours with CG storage discharge in the EOE model. This is also a consequence of a possible underestimation of the firmness factor of the CG storage technology (Table 2).
- New CG PV investments only take place in the EOE market model option whenever the access tariff is fully energy base one. This, in combination with the high BMDG PV investments in all scenarios, shows that the economies of scale in centralized PV investment costs do not offset the effect of the access-tariff costs (at least for the costs scenario used in this work). This also suggests the very significant impact that self-generation may have in the coming years in the evolution of the power sector, as many reports already highlight [39].

The next subsection analyzes how energy market prices are related to both CG and BMDG investments.

3.2.4. Energy market prices and hourly energy market price

As a function of the weight of the power-based tariff term, Figure 6 shows the yearly average value of hourly energy prices $\lambda_{h,y}^{E}$ (JECE energy price) and hourly capacity prices $\lambda_{h,y}^{C}$ (JECE capacity price) for the JECE market, the yearly average value of the hourly energy market price for the EOE market (dual variable of the generation-demand constraint, noted as Market price EOE), and the yearly average value of the total hourly supplied energy market price for the JECE market, computed as the sum of the hourly energy price $\lambda_{h,y}^{S}$ and the hourly capacity price $\lambda_{h,y}^{L}$.

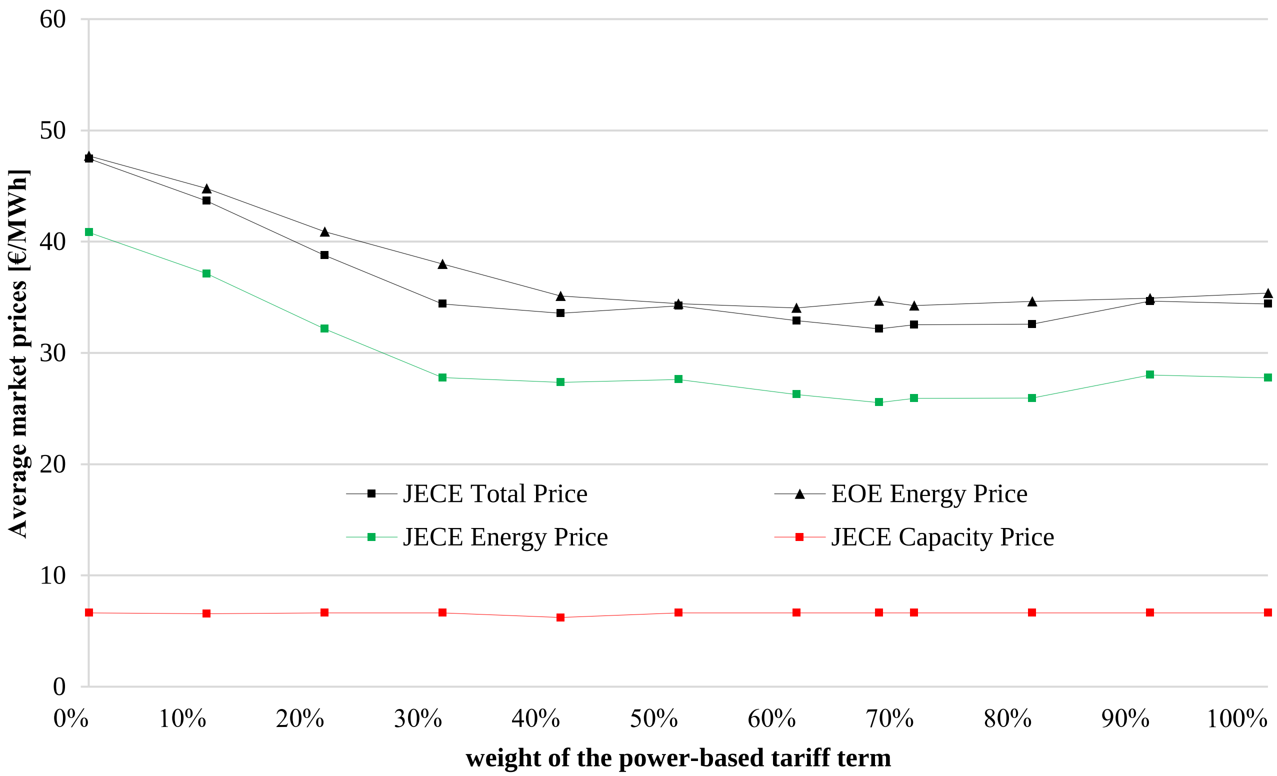

Figure 6: Average market prices in 2039 per weight of the power-based tariff term

Also, Figure 7 shows the distribution of these total hourly energy market prices for both market models (sum of $\lambda^{E}_{h,y}$ and $\lambda^{C}_{h,y}$ in the case of JECE and only $\lambda^{E}_{h,y}$ in the case of EOE) using classical boxplots.

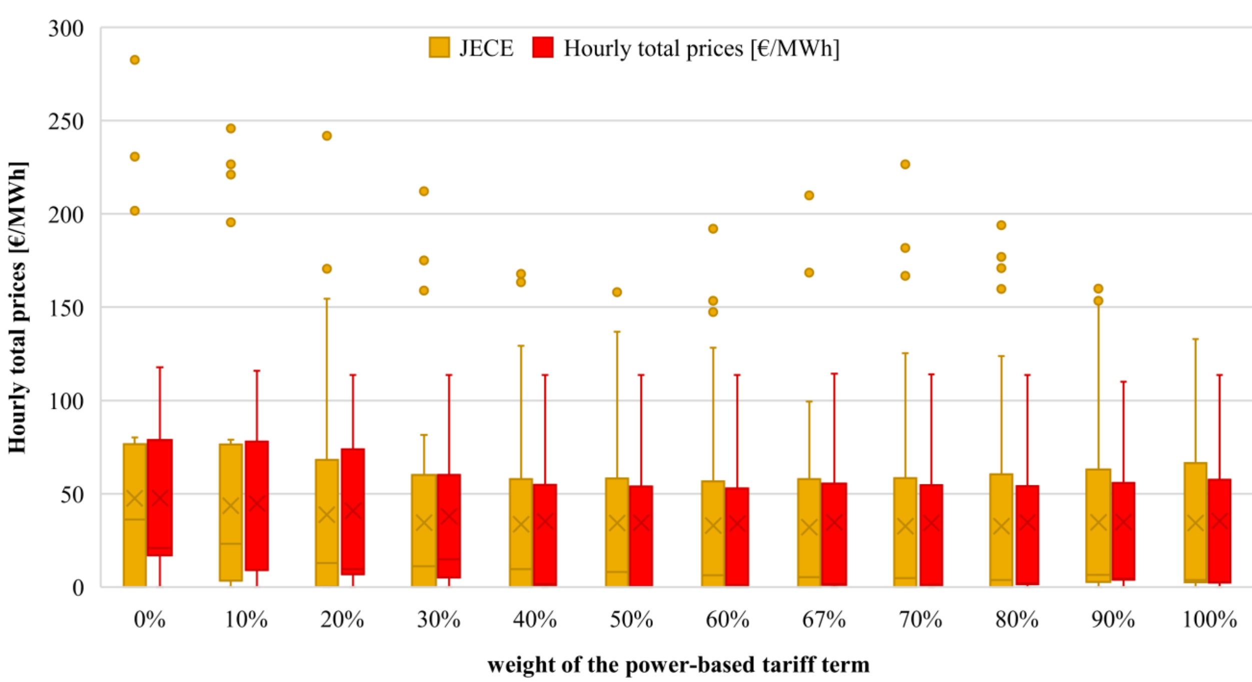

Figure 7: Hourly market price distribution in 2039 per weight of the power-based tariff term

Figures 6 & 7 show that:

- The yearly average of capacity market prices $\lambda^{C}_{h,y}$, for the JECE market, remains almost constant among all tariff scenarios. This is straightforward since OCGT is the marginal technology in all scenarios in the capacity market (limited to those hours where the capacity constraint (4) becomes active).
- Total yearly average energy prices are higher in the EOE model than in the JECE model. The reason behind this is that higher CG storage investments in the EOE model (see Figure 5) internalize their investment costs in the energy market price. However, EOE also leads to a cleaner mix (i.e. with lower CO_2 emissions due to a lower thermal generation), once again possibly due to the underestimation of the firmness capacity of BMDG storage.
- However, the price distribution is wider in the JECE model than in the EOE model. This is because $\lambda^{C}_{h,y}$ of the JECE model is positive only during the few peak hours, causing critical-peak pricing ([40]), being null during all the other hours of the year, providing a better economic signal of the system's capacity scarcity.
- Below 40%, the lower the weight of the power-based tariff term is, that is, the higher the weight of the energy base tariff term is, the higher the average market price is, and the larger the variability of the hourly market prices is. This is due to the also higher average value and larger variability of the grid demand in those conditions(see Figure 8 in the next subsection), as a result of lower BMDG investments (see Figure 3).

To explain these variabilities in prices, the next subsection shows the distribution of the hourly demand.

3.2.5. Hourly demand distribution

Figure 8 shows the distribution of the hourly demand for each scenario.

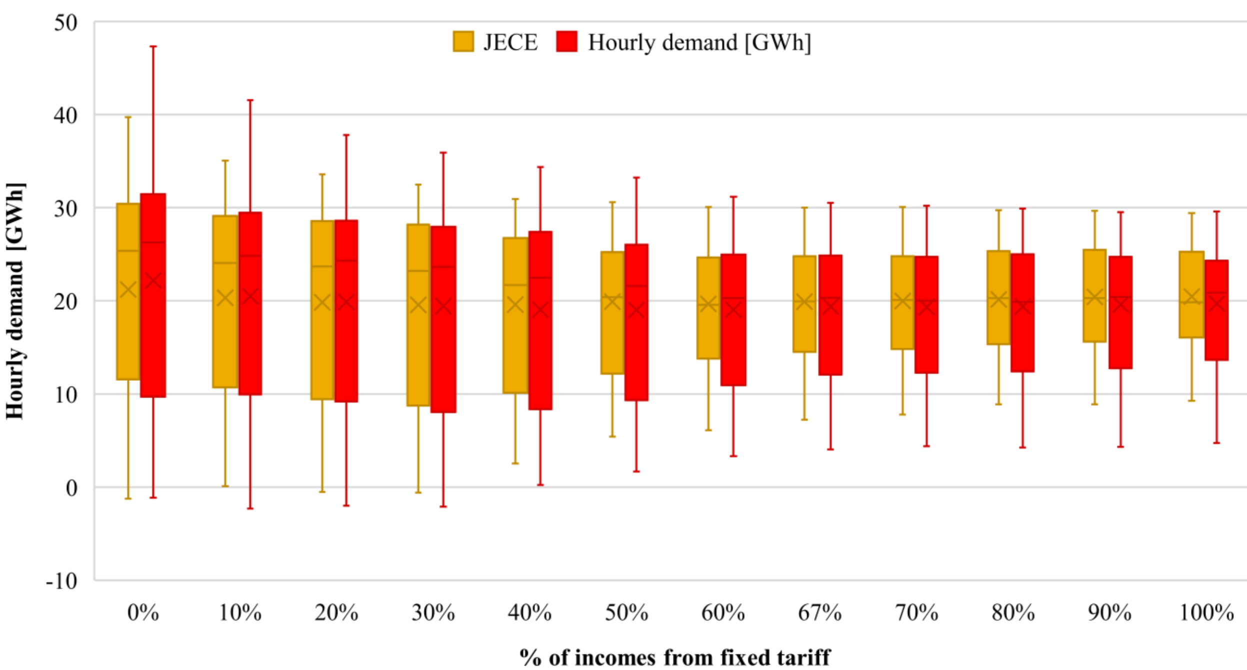

Figure 8: Hourly demand distribution in 2039 per fixed tariff levels

Figure 8 shows that:

- The lower the weight of the power-based tariff term is, the larger the variability of the hourly grid demand profiles is, since incentives for load shifting are lower. It results in lower DG storage investments and higher CG storage investments.
- Peak demand is higher in the EOE market model option than in the JECE one except for weights of the power-based tariff term above 70%, for which the peak grid demands for both market model options are almost the same. It shows again that the power-based tariff term represents an incentive to reduce peak consumption, coherently with Figure 4, in which of could be observed that the contracted power (peak grid demand per customer) was almost equal in both market model options for high weights of the power-based tariff term.
- For a low weight of the power-based tariff term (below 30%) results even show negative total grid demand for a few hours. This means that, for those hours, GENCOs are consuming and storing energy the customers are producing. These extreme scenarios occur due to the high DG PV investments combined with the low DG storage investments (see Figure 3), and the high CG storage investments (see Figure 5).

4. Conclusions

This paper proposes a Nash equilibrium mathematical model to address the joint centralized and behind-the-meter distributed generation expansion planning and operation, with an explicit representation of both energy and capacity markets. The paper proofs that this complex equilibrium model can be solved as an equivalent cost-minimization problem, which significantly reduces the computational burden and makes much easier to include new additional constraints whenever required. The model computes energy and capacity prices and proposes a design for setting the consumer hourly capacity payments. Exogenous access-tariffs (energy and capacity terms) are set to recover a fixed amount of regulatory and policy-related costs in the system, and also a target margin above the grid demand peak, which is fulfilled with firm capacity to ensure a certain level of security of supply.

Results of the energy and capacity market model are compared with an energy-only market approach for a real-sized case study, the Spanish system. The case studies analyzed show that:

- The economic signal sent by the capacity market to the consumers incentivizes collaborating in reducing the total system (grid) peak demand. However, this economic signal is only efficient for weights of the power-based term of the access tariff below 70% of the full amount collected by the access tariff. For higher values, the impact of the capacity market on the grid peak load dilutes and the system peak demand is similar to that of an energy-only market.

- A market design based on both energy and capacity markets leads to a power system with less variability in the total grid demand and with lower average market prices, due to higher incentives to invest in DG storage facilities to be used for peak-shaving. Energy-only markets, instead, lead to more frequent and spiky episodes of scarcity hours with very high prices. the peaker generation technologies need these high prices to recover their investment costs.

- In comparison with an energy-only market, the joint energy and capacity market design leads to higher investments in peaker technologies and lower utility-scale storage investments. However, these results are heavily conditioned by the firmness factors, in particular the assumption of null firmness factors for utility-scale storage technologies. This assumption, based on the publicly available data of the Spanish transmission system operator, the so-called availability factors, corresponds to storage technologies based on batteries not able to generate at full capacity for more than 2 hours, providing indeed very poor support to the security of supply of the system.

Future work should consider other kinds of storage technologies (batteries with higher energy/power ratios, pumping, power to gas, …), which may nuance this last conclusion. The authors are also currently working on upgrading the model with the formulation of an upper-level layer to endogenously optimize the structure of the two terms access-tariffs applied to consumers, such that the total amount of money collected from the resulting structure is large enough to recover the system's regulated costs (such as transmission and distribution network costs) and policy-driven costs (such for instance support to renewable sources related costs).

Annex I

The following equations are the Lagrange functions[8] of (6) subject to (10) (for $s \in G$) and **¡Error! No se encuentra el origen de la referencia.** subject to (1) and (10) (for $s \in C$).

$$\begin{aligned}
&\ell_g(q,p,\lambda^{PQ},\lambda^S,\lambda^L) \\
&\quad = \sum_y (1+W)^{-y} \\
&\quad \cdot \left[-\sum_{t,h} \left(\left(\lambda^S_{h,y} - VC_{t,g,y}\right) \cdot q_{t,g,h,y} \right) - \sum_t \left(\left(\lambda^L_y \cdot DI_t - IC_{t,g,y}\right) \cdot p_{t,g,y} \right) \right. \\
&\quad \left. + \sum_{t,h} \lambda^{PQ}_{t,g,h,y} \cdot \left(q_{t,g,h,y} - p_{t,g,y}\right) \right]
\end{aligned}$$

$$\begin{aligned}
&\ell_c(q,p,cp,dq,eq,\lambda^S,\lambda^L,\lambda^{PQ},\lambda^{CB},\lambda^{CP}) \\
&\quad = \sum_y (1+W)^{-y} \\
&\quad \cdot \left[TP_{c,y} \cdot cp_{c,y} + \sum_t IC_{t,c,y} \cdot p_{t,c,y} \right. \\
&\quad + \sum_h \left[\left(\lambda^S_{h,y} + \lambda^L_{h,y} + TV_{c,y}\right) \cdot dq_{c,h,y} - \left(\lambda^S_{h,y} + \lambda^L_{h,y} - TC_{c,y}\right) \cdot eq_{c,h,y} \right] \\
&\quad + \sum_{t,h} \lambda^{PQ}_{t,c,h,y} \cdot \left(q_{t,c,h,y} - p_{t,c,y}\right) + \sum_{h,y} \lambda^{CB}_{c,h,y} \cdot \left(\sum_t q_{t,c,h,y} + dq_{c,h,y} - D_{c,h,y} - eq_{c,h,y} \right) \\
&\quad \left. + \sum_h \lambda^{CP}_{c,h,y} \cdot \left(dq_{c,h,y} + eq_{c,h,y} - cp_{c,y}\right) \right]
\end{aligned} \tag{15}$$

[8]These Lagrangean functions consider that all constraints have been divided in both sides by the discount rate $(1+W)^y$.

Their KKT conditions are:

$$
\begin{aligned}
&\frac{\partial \ell_g}{\partial q_{t,g,h,y}} = VC_{t,g,y} - \frac{\partial \lambda^E_{h,y}}{\partial q_{t,g,h,y}} \cdot \left(\sum_{t\prime} q_{t\prime,g,h,y} \right) - \lambda^E_{h,y} + \lambda^{PQ}_{t,g,h,y} = 0 \\
&\frac{\partial \ell_c}{\partial q_{t,c,h,y}} = \lambda^{PQ}_{t,c,h,y} + \lambda^{CB}_{c,h,y} = 0 \\
&\frac{\partial \ell_c}{\partial cp_{c,y}} = TP_{c,y} - \sum_h \lambda^{CP}_{c,h,y} = 0 \\
&\frac{\partial \ell_c}{\partial dq_{c,h,y}} = \frac{\partial \left(\lambda^E_{h,y} + \lambda^C_{h,y} \right)}{\partial dq_{c,h,y}} \cdot \left(eq_{c,h,y} - dq_{c,h,y} \right) + \left(\lambda^E_{h,y} + \lambda^C_{h,y} + TV_{c,y} \right) \\
&\qquad + \lambda^{CB}_{c,h,y} + \lambda^{CP}_{c,h,y} = 0 \\
&\frac{\partial l_c}{\partial eq_{c,h,y}} = \frac{\partial \left(\lambda^E_{h,y} + \lambda^C_{h,y} \right)}{\partial eq_{c,h,y}} \cdot \left(eq_{c,h,y} - dq_{c,h,y} \right) - \left(\lambda^E_{h,y} + \lambda^C_{h,y} - TV_{c,y} \right) \\
&\qquad - \lambda^{CB}_{c,h,y} + \lambda^{CP}_{c,h,y} = 0 \\
&\frac{\partial \ell_g}{\partial p_{t,g,y}} = IC_{t,g,y} - \frac{\partial \lambda^C_y}{\partial p_{t,g,y}} \cdot \left(\sum_{t\prime} DI_{t\prime} \cdot p_{t\prime,g,y} \right) - DI_t \cdot \lambda^C_{h,y} \\
&\qquad - \sum_h \lambda^{PQ}_{t,g,h,y} = 0 \\
&\frac{\partial \ell_c}{\partial p_{t,c,y}} = IC_{t,c,y} - \sum_h \lambda^{PQ}_{t,c,h,y} = 0 \\
&\lambda^{CB}_{c,h,y} \perp \sum_t q_{t,c,g,y} - eq_{c,h,y} + dq_{c,h,y} - D_{c,h,y} = 0 \\
&0 \le \lambda^{PQ}_{t,s,h,y} \perp q_{t,s,h,y} - p_{t,s,y} \le 0, s \in C \cup G \\
&0 \le \lambda^{CP}_{c,h,y} \perp dq_{c,h,y} + eq_{c,h,y} - cp_{c,y} \le 0 \\
&\lambda^E_{h,y} \perp \sum_{t,g} q_{t,g,h,y} + \sum_c \left(eq_{c,h,y} - dq_{c,h,y} \right) = 0 \\
&\lambda^C_y \perp \sum_{t,g} DI_t \cdot p_{t,g,y} - (1 + CI) \cdot d^M_y = 0 \\
&0 \le \lambda^C_{h,y} \perp \sum_c dq_{c,h,y} - \sum_c eq_{c,h,y} - d^M_y \le 0
\end{aligned}
\tag{16}
$$

It can be checked that the KKT conditions of (14) subject to (1), (2), (4), (5), and (10) are the same as the previous ones, taking into account (11) and (13) (and assuming null the crossed conjectures). Moreover, deriving the Lagrange function of (14) with respect to $d^M{}_y$ then:

$$
\sum_h \lambda^C_{h,y} = (1 + CI) \cdot \lambda^C_y \tag{17}
$$

which implies, according to (5) and since $\lambda^C{}_{h,y} = 0$ for $h \in H^M{}_y$:

$$\sum_{h,y} d_y^M \cdot \lambda_{h,y}^C = \sum_y d_y^M \cdot (1 + CI) \cdot \lambda_y^C \Leftrightarrow$$
$$\sum_{c,h,y} \left(dq_{c,h,y} - eq_{c,h,y}\right) \cdot \lambda_{h,y}^C = \sum_{t,g,y} DI_t \cdot p_{t,g,y} \cdot \lambda_y^C \tag{18}$$

This proves that (8) holds true.

Acknowledgments

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