XI Congreso de Ingeniería de Organización
International Conference on Industrial Engineering and Industrial Management
Madrid. September 5th-7th 2007

MATHEMATICAL PROGRAMMING APPROACH TO UNDERGROUND TIMETABLING PROBLEM FOR MAXIMIZING TIME SYNCHRONIZATION

ANDRES RAMOS
MARIA TERESA PENA
ANTONIO FERNANDEZ
PALOMA CUCALA
INSTITUTO DE INVESTIGACION TECNOLOGICA. ESCUELA TECNICA SUPERIOR DE INGENIERIA
UNIVERSIDAD PONTIFICIA COMILLAS

Key words: *Train timetabling problem, train scheduling, energy saving.*

1. Introduction

Underground transportation is crucial in big modern cities as a way to achieve a clean, rapid and massif transport. While in peak hours the first objective is to move as many people as possible increasing the train frequency, in off-peak hours other considerations can be taken into account. Energy consumption should be an important issue in the design of train timetables in off-peak hours. Energy saving can be obtained by using regenerative brakes and allowing to synchronize the speed-up of a train exiting a station and the slow-down of a train arriving to another station connected to the same electric section. The arriving train generates electricity that is consumed by the departing train. In fact, given the high frequency of trains in peak hours it is more probable the coincidence of these two processes and, at the same time, is more difficult to change the train schedules by the same reason. Thus the first step in energy saving by changing the off-peak train timetable is maximizing the time overlap between different trains in the same or in different stations connected to the same electrical substation.

By nature the train timetabling problem is highly combinatorial, tightly constrained and very difficult to solve. Two main approaches have been followed by the researchers for solving the problem. One based on mathematical programming techniques. See the papers Caprara *et al.* (2001) and Caprara *et al.* (2002) for the use of Lagrangian relaxation combined with subgradient optimization. See Bussieck (1997) and Nielsen (2006) for mixed integer programming formulations solved directly. In this paper we also use this approach. The other approaches are based on metaheuristic techniques. See Godwin (2006), where they resort to genetic algorithms. Recently, a hybrid approach that combines an evolutionary algorithm for obtaining an initial solution for the optimization problem has been proposed, see Semet (2005), and also constraint programming as solution environment, see Rodriguez (2007).

The paper is organized as follows. In section 2 it is presented the rational of the model. Then, the mathematical formulation of the optimization problem is stated in section 3. In section 4, we develop a case study corresponding to line 1 of Metro de Madrid. Finally, some conclusions are summarized and some future extensions are suggested in the last section.

2. Model description

The model presented here is a particular case of train timetabling problem. Its purpose is maximizing the overlapping time between speed-up and slow-down actions of all the trains circulating at any time and located in the same electrical section. As said in the introduction, the model is applied only to trains running during off-peak hours (after 23 h) because their schedule can be more easily changed. Moreover, for night trains schedules can be observed almost strictly due to lack of incidences. This model is a useful tool to define the off-peak timetables where energy saving can be an important goal in train scheduling. An experience in the Rome (Italy) underground has reported an energy saving of 15% without a synchronization objective, see Adinolfi (1998).

The initial schedule is taken as given and the model maximizes the coincidence time while satisfying several operating constraints in order to obtain an implementable timetable.

It is stated as a mixed integer (MIP) optimization problem. The objective function is to maximize the overlapping time between trains that arrive and depart from the same station of from different stations connected to the same electrical substation. The constraints include upper bounds on changes in the current timetable with respect to an initial schedule, while keeping the total travel time of each train, and the computation of the coincidence time between trains. The detection of the overlapping condition requires binary variables and, therefore, integrality conditions, which make the problem very difficult to solve.

Several uses of interest can be devised when solving the optimization problem: i) evaluation of the overlapping time for the initial timetable, ii) maximizing overlapping time, but keeping the train trip time in order to drive at economical speed, similarly to the advertised timetable that exclusively determines departure times, iii) overlapping time when arrival and departure times can be optimized.

3. Mathematical formulation

3.1. Indices

i train. Trains are supposed to do just a round trip from the beginning to the ending station and then back.

j platform (for example, northbound and southbound) of an underground station. $j = 1,, J$, being 1 the departure platform of the head station and J the opposite platform of the head station.

3.2. Parameters

The following data are supposed to be known in advance and correspond to the initial timetable, to intervals of the slow-down and speed-up processes, and to some adjustment parameters that avoid dramatic changes in the final schedule. We use lower case letters to define the parameters.

a_{ij}, d_{ij} initial arrival and departure times of train *i* at platform *j* [s]

sd, su slow-down and speed-up times of any train at any platform[1] [s]

Δs_j, ∇s_j, maximum and minimum changes in stopping time at platform *j* [s]

Δt_j, ∇t_j, maximum and minimum changes in travelling time at platform *j* [s]

Δtt maximum increment in total trip time for any train [s]

$p_{jj'}$ penalty factor introduced to consider somehow the loss in the electricity transferred between trains at different platforms *j* and *j'* although both belong to the same electrical section [p.u.]. If two platforms belong to different electrical sections $p_{jj'} = 0$.

3.3. Variables

The variables of the optimization problem are written in capital and Greek letters and correspond to the following ones:

A_{ij}, D_{ij} arrival and departure times of train *i* at platform *j* [s]

$\delta_{iji'j'}$ binary variable that indicates whether there is or not (1/0, respectively) coincidence between the slow-down interval of train *i* at platform *j* and the speed-up interval of train *i'* at platform *j'*

$T_{iji'j'}$ overlapping time between the slow-down and speed-up intervals of train *i* at platform *j* and train *i'* at platform *j'*, respectively [s]

B_{ij}, C_{ij} change in arrival and departure times of train *i* at platform *j* with respect to the initial timetable [s]

[1] They can easily be particularized for each platform and even train type to take into consideration their specific characteristics.

3.4. Constraints

The following constraints take into account the operating conditions of the trains.

— Change in the stopping time with respect to the initial schedule for each train i at platform j has to be bounded by the corresponding bounds

$$\nabla s_j \le (D_{ij} - A_{ij}) - (d_{ij} - a_{ij}) \le \Delta s_j \quad \forall ij \qquad [1]$$

The stopping time of any train at the terminal station is considered to take a constant time. Therefore, this constraint is not formulated at the terminal station of the line. Each time a train departs from the head station is considered a new train.

— Change in the travelling time for each train i at platform j with respect to the initial schedule has to be bounded by the corresponding bounds

$$\nabla t_j \le (A_{ij} - D_{ij-1}) - (a_{ij} - d_{ij-1}) \le \Delta t_j \quad \forall ij \qquad [2]$$

The platform change of the train at the terminal station is considered to take a constant time. Therefore, this constraint is not formulated for the terminal station of the line.

— Change in the total trip time for each train i and in each way with respect to the initial schedule has to be bounded by the corresponding bounds

$$\begin{aligned} &(A_{i\,J/2} - D_{i1}) - (a_{i\,J/2} - d_{i1}) \le \Delta tt \quad \forall i \\ &(A_{iJ} - D_{i\,J/2+1}) - (a_{iJ} - d_{i\,J/2+1}) \le \Delta tt \quad \forall i \end{aligned} \qquad [3]$$

One way is from the departure platform at the head station to same side platform at the terminal station, $J/2$, and the other way is from other side platform at the terminal station $J/2+1$ to the opposite platform at the head station J,

— Computation of overlapping time

Before writing the constraints that allow the computation of the overlapping time let us describe the different possibilities of train coincidence. Let us define $A^-_{ij} = A_{ij} - sd$ the beginning of the slow-down process before arriving to a platform $D^+_{i'j'} = D_{i'j'} + su$ the end of the speed-up process after departure of a platform. The six combinations and their overlapping time are presented in the following table, where departure times are in black colour and arrival times are in blue.

Case	Sequence		Overlapping time
1	$D_{i'j'}, A^-_{ij}, A_{ij}, D^+_{i'j'}$	[[]]	$A_{ij} - A^-_{ij} = sd$
2	$A^-_{ij}, D_{i'j'}, D^+_{i'j'}, A_{ij}$	[[]]	$D^+_{i'j'} - D_{i'j'} = su$
3	$D_{i'j'}, A^-_{ij}, D^+_{i'j'}, A_{ij}$	[[]]	$D^+_{i'j'} - A^-_{ij}$
4	$A^-_{ij}, D_{i'j'}, A_{ij}, D^+_{i'j'}$	[[]]	$A_{ij} - D_{i'j'}$
5	$D_{i'j'}, D^+_{i'j'}, A^-_{ij}, A_{ij}$	[][]	0
6	$A^-_{ij}, A_{ij}, D_{i'j'}, D^+_{i'j'}$	[][]	0

For example, there is no coincidence if a train begins the slow-down process after the speed-up process of another train ($A^-_{ij} \ge D^+_{i'j'}$, case [5]) or if a train departs after the arrival of another train ($D_{i'j'} \ge A_{ij}$, case [6]). These cases can be modelled as a logical implication

$$A^-_{ij} \ge D^+_{i'j'} \text{ or } D_{i'j'} \ge A_{ij} \Rightarrow \delta_{iji'j'} = 0 \qquad [4]$$

being $\delta_{iji'j'}$ the binary variable that indicates the coincidence condition. $\delta_{iji'j'} = 1$ means coincidence.

This implication can be modelled by the linear constraints

$$\begin{aligned} &A^-_{ij} - D^+_{i'j'} \le M(1 - \delta_{iji'j'}) \quad \forall iji'j' \\ &D_{i'j'} - A_{ij} \le M(1 - \delta_{iji'j'}) \quad \forall iji'j' \end{aligned} \qquad [5]$$

being $M = \max\left(\left|d_{i'j'} - a_{ij} - \Delta s_j + \nabla s_j\right|\right) + su + sd$ an upper bound of the constraint.

In the other cases [1 to 4] and if we define $B_{iji'j'}$ and $E_{iji'j'}$ as the beginning and end of overlapping time between the slow-down interval of train i at platform j and speed-up internal of train i' at platform j', the overlapping time can be calculated as

$$\begin{aligned} &B_{iji'j'} = \max(D_{i'j'}, A^-_{ij}) && \forall iji'j' \\ &E_{iji'j'} = \min(D^+_{i'j'}, A_{ij}) && \forall iji'j' \\ &T_{iji'j'} \le M'\delta_{iji'j'} && \forall iji'j' \\ &T_{iji'j'} \le (E_{iji'j'} - B_{iji'j'}) + M(1 - \delta_{iji'j'}) && \forall iji'j' \end{aligned} \qquad [6]$$

being the maximum overlapping time the

slow-down or speed-up time of any train, $M' = \min(su, sd)$. In fact, we can disregard the auxiliary variables $B_{iji'j'}$ and $E_{iji'j'}$ and formulate the constraint as

$$
\begin{aligned}
&T_{iji'j'} \le M'\delta_{iji'j'} && \forall iji'j' \\
&T_{iji'j'} \le D^{+}_{i'j'} - D_{i'j'} + M(1-\delta_{iji'j'}) && \forall iji'j' \\
&T_{iji'j'} \le D^{+}_{i'j'} - A^{-}_{ij} + M(1-\delta_{iji'j'}) && \forall iji'j' \\
&T_{iji'j'} \le A_{ij} - D_{i'j'} + M(1-\delta_{iji'j'}) && \forall iji'j' \\
&T_{iji'j'} \le A_{ij} - A^{-}_{ij} + M(1-\delta_{iji'j'}) && \forall iji'j'
\end{aligned}
\qquad [7]
$$

$$
\begin{aligned}
&T_{iji'j'} \le M'\delta_{iji'j'} && \forall iji'j' \\
&T_{iji'j'} \le su\,\delta_{iji'j'} && \forall iji'j' \\
&T_{iji'j'} \le D^{+}_{i'j'} - A^{-}_{ij} + M(1-\delta_{iji'j'}) && \forall iji'j' \\
&T_{iji'j'} \le A_{ij} - D_{i'j'} + M(1-\delta_{iji'j'}) && \forall iji'j' \\
&T_{iji'j'} \le sd\,\delta_{iji'j'} && \forall iji'j'
\end{aligned}
\qquad [8]
$$

It can be observed that the third and fourth equations of the set [8] and the condition of non negative overlapping time $T_{iji'j'} \ge 0$ turn superfluous equations [5]. In the same way, the second and fifth equations of set [8] can be substituted by the first equation of this set.

$$
\begin{aligned}
&T_{iji'j'} \le M'\delta_{iji'j'} && \forall iji'j' \\
&T_{iji'j'} \le D^{+}_{i'j'} - A^{-}_{ij} + M(1-\delta_{iji'j'}) && \forall iji'j' \\
&T_{iji'j'} \le A_{ij} - D_{i'j'} + M(1-\delta_{iji'j'}) && \forall iji'j'
\end{aligned}
\qquad [9]
$$

— Finally, a set of equations are added to avoid changes in the timetable that do not improve the overlapping time

$$
\begin{aligned}
&-B_{ij} \le A_{ij} - a_{ij} \le B_{ij} && \forall ij \\
&-C_{ij} \le D_{ij} - d_{ij} \le C_{ij} && \forall ij
\end{aligned}
\qquad [10]
$$

where variables B_{ij} and C_{ij} correspond to changes in arrival and departure times, respectively, with respect to the initial timetable. Their sum is introduced in the objective function with a very small penalty ε.

3.5. Objective function

The objective function maximizes the total overlapping time

$$
\max \sum_{iji'j'} p_{jj'} T_{iji'j'} - \varepsilon \sum_{ij} \left(B_{ij} + C_{ij} \right) \qquad [11]
$$

3.6. Mathematical problem

The MIP optimization problem that maximizes the total overlapping time between the slow-down and speed-up processes of different trains can be stated as

$$
\begin{aligned}
&\max \sum_{iji'j'} p_{jj'} T_{iji'j'} - \varepsilon \sum_{ij} \left(B_{ij} + C_{ij} \right) \\
&\nabla s_j \le (D_{ij} - A_{ij}) - (d_{ij} - a_{ij}) \le \Delta s_j && \forall ij \\
&\nabla t_j \le (A_{ij} - D_{ij-1}) - (a_{ij} - d_{ij-1}) \le \Delta t_j && \forall ij \\
&(A_{i\,J/2} - D_{i1}) - (a_{i\,J/2} - d_{i1}) \le \Delta tt && \forall i \\
&(A_{ij} - D_{i\,J/2+1}) - (a_{ij} - d_{i\,J/2+1}) \le \Delta tt && \forall i \\
&T_{iji'j'} \le M'\delta_{iji'j'} && \forall iji'j' \\
&T_{iji'j'} \le D^{+}_{i'j'} - A^{-}_{ij} + M(1-\delta_{iji'j'}) && \forall iji'j' \\
&T_{iji'j'} \le A_{ij} - D_{i'j'} + M(1-\delta_{iji'j'}) && \forall iji'j' \\
&-B_{ij} \le A_{ij} - a_{ij} \le B_{ij} && \forall ij \\
&-C_{ij} \le D_{ij} - d_{ij} \le C_{ij} && \forall ij \\
&A_{ij}, D_{i'j'}, T_{iji'j'}, B_{ij}, C_{ij} \ge 0, \delta_{iji'j'} \in \{0,1\}
\end{aligned}
\qquad [12]
$$

		$I = 14, J = 54$
Contraints	$8IJ + 2I + 3I^2J^2$	1720684
Continuous variables	$4IJ + I^2J^2$	1146096
Binary variables	I^2J^2	571536

The size of the problem is parameterized in this table and estimated for $I = 14$ night trains, and $J = 54$ platforms (corresponding to 27 stations).

To avoid the curse of dimensionality in the size of the problem the possible combinations between different trains at platforms can be substantially reduced by dealing only with those trains that are relative close in the original timetable, $iji'j' \in c(i.j, i', j')$, being $c(i.j, i', j')$ the set of close trains. For example, in the case study presented in the following section the size of the problem is approximately 7,700 constraints, 4,200 continuous variables and 600 binary variables very far from the previous estimation. The set of close trains is determined by the model given a scalar specified by the user.

3.7. Implementation

The model has been written in GAMS, see Brooke (2005), and solved by CPLEX 10.1, see ILOG, under a PC at 1.83 GHz with 1 GB of RAM memory running the Microsoft Windows XP operating system. A Microsoft Excel interface has been used for input data and output results.

In the following table are presented some results of the mathematical problem for different maximum solution times.

It can be observed that the improvement in the MIP optimal solution is very low with respect to the maximum solution time. The difficulty in solving a MIP problem is somehow measured by the relative tolerance or integrality gap, which in this case is high. The number of iterations and explored nodes are proportional to the solution time.

4. Case study

This train timetabling model has been tested with a realistic case corresponding to line 1 of Metro de Madrid. As mentioned in the introduction, the optimization problem may be used for:

— Evaluation of the overlapping time for the initial arrival and departure times

  The initial timetable had 0 seconds of overlapping time.

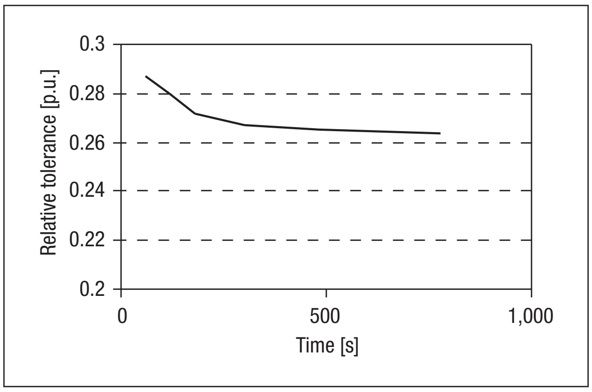
— Maximizing overlapping time, but keeping the train trip time in order to drive at economical speed, similarly to the advertised timetable that exclusively determines departure times

  For this case the overlapping time reached 1.30 hours (within a solution time of 60 s).

— Optimization of overlapping time when arrival and departure times are optimized

  If arrival times as well as departure times can be scheduled, the overlapping time can be slightly improved up to 1.36 hours (within a solution time of 60 s).

Figure 1
Evolution of the relative tolerance with respect to the maximum solution time



These results show that time coincidence can be increased dramatically and, therefore, energy saving by train synchronization. Although the MIP solution obtained in a certain clock time can not be proved to be optimal it represents a dramatic improvement in the objective function with respect to the overlapping time of the initial timetable. Besides, much of the overlapping time can be achieved with no modifications of the current advertised timetables, only modifying the internal operation of the trains without modifying the public perception.

The following coloured table show where the changes in the timetable have been made for the second case mentioned before. Because of the huge amount of data, only the stations connected to a given substation are shown. The trains that simultaneously overlap when slow down and speed up are labelled with the same colour.

The following table shows how much overlapping ti-

	60 s	**120 s**	**180 s**	**300 s**	**480 s**	**780 s**
MIP solution [s]	4909.645	4932.533	4957.533	4972.661	4972.587	4972.661
LP Relaxation [s]	6320.811	6312.254	6305.047	6298.265	6290.912	6285.199
Relative Tolerance [p.u.]	0.287427	0.279718	0.271811	0.266578	0.265119	0.263951
Iterations	136564	283620	452945	708080	1171531	1886725
Nodes	18701	38101	60701	95301	157401	247801

Table 1

Coincident trains when the timetable is calculated optimizing only departure times

Calculated Schedule		Stations											
Trains	Events	JA1	S1	TM1	AM1	AT1	AR1	AR2	AT2	AM2	TM2	S2	JA2
N1	Arrival	23:19:00	23:19:55	23:21:50	23:22:45	23:24:40	23:25:35	24:06:40	24:07:35	24:08:30	24:10:40	24:12:35	24:14:00
	Departure	23:19:10	23:20:05	23:22:00	23:22:55	23:24:50	23:25:45	24:06:50	24:07:45	24:08:55	24:10:50	24:13:15	24:14:10
N2	Arrival	23:26:00	23:27:55	23:28:49	23:30:45	23:31:39	23:32:34	24:14:05	24:15:00	24:16:55	24:17:50	24:19:45	24:20:40
	Departure	23:26:10	23:28:05	23:29:00	23:30:55	23:31:50	23:33:08	24:14:15	24:15:10	24:17:05	24:18:00	24:19:55	24:20:50
N3	Arrival	23:34:28	23:35:24	23:37:18	23:38:13	23:40:09	23:41:04	24:28:41	24:30:06	24:32:01	24:32:56	24:34:51	24:35:46
	Departure	23:34:39	23:35:34	23:37:29	23:38:24	23:40:19	23:41:19	24:29:21	24:30:15	24:32:10	24:33:06	24:35:01	24:35:55
N4	Arrival	23:41:29	23:43:24	23:44:19	23:46:14	23:47:09	23:48:04	24:43:48	24:45:13	24:47:08	24:48:03	24:49:58	24:50:53
	Departure	23:41:39	23:43:34	23:44:29	23:46:24	23:47:19	23:48:19	24:44:27	24:45:23	24:47:18	24:48:13	24:50:08	24:51:03
N5	Arrival	23:49:59	23:50:55	23:52:55	23:53:49	23:55:45	23:56:40	24:57:58	24:59:23	25:01:30	25:02:25	25:04:20	25:05:15
	Departure	23:50:09	23:51:10	23:53:05	23:53:59	23:55:55	23:56:50	24:58:38	24:59:45	25:01:40	25:02:35	25:04:30	25:05:25
N6	Arrival	23:56:40	23:58:35	23:59:30	24:01:25	24:02:20	24:03:20	25:13:15	25:14:40	25:16:36	25:17:31	25:19:26	25:20:21
	Departure	23:56:50	23:58:45	23:59:40	24:01:35	24:02:35	24:03:30	25:13:55	25:14:50	25:16:45	25:17:41	25:19:36	25:20:30
N7	Arrival	24:04:20	24:05:15	24:07:10	24:08:05	24:10:00	24:11:10	23:20:55	23:22:15	23:23:10	23:25:05	23:27:00	23:28:20
	Departure	24:04:29	24:05:25	24:07:20	24:08:15	24:10:25	24:11:50	23:21:30	23:22:25	23:23:20	23:25:15	23:27:35	23:28:30
N8	Arrival	24:11:10	24:13:35	24:14:30	24:16:25	24:17:20	24:18:15	23:28:00	23:29:13	23:31:09	23:32:03	23:33:58	23:34:54
	Departure	24:11:50	24:13:45	24:14:40	24:16:35	24:17:30	24:18:50	23:28:29	23:29:24	23:31:19	23:32:14	23:34:09	23:35:04
N9	Arrival	24:26:50	24:28:45	24:29:40	24:31:35	24:32:30	24:33:25	23:36:18	23:37:42	23:38:37	23:40:34	23:42:28	23:43:50
	Departure	24:27:00	24:28:55	24:29:49	24:31:44	24:32:40	24:33:34	23:36:58	23:37:53	23:38:49	23:40:44	23:43:04	23:44:00
N10	Arrival	24:41:57	24:43:52	24:44:47	24:46:42	24:47:37	24:48:32	25:28:30	25:29:47	25:31:42	25:32:37	25:34:32	25:35:27
	Departure	24:42:06	24:44:01	24:44:57	24:46:52	24:47:47	24:48:42	25:29:01	25:29:57	25:31:52	25:32:47	25:34:42	25:35:37
N11	Arrival	24:56:08	24:58:03	24:58:58	25:00:53	25:01:55	25:02:55	25:49:50	25:51:15	25:52:10	25:54:14	25:56:09	25:57:04
	Departure	24:56:18	24:58:13	24:59:08	25:01:10	25:02:10	25:03:35	25:50:30	25:51:25	25:52:29	25:54:24	25:56:18	25:57:14
N12	Arrival	25:11:25	25:13:20	25:14:15	25:16:10	25:17:05	25:18:00	23:43:34	23:44:44	23:46:39	23:47:34	23:49:28	23:50:25
	Departure	25:11:35	25:13:30	25:14:24	25:16:19	25:17:15	25:18:09	23:43:59	23:44:54	23:46:49	23:47:44	23:49:39	23:50:35
N13	Arrival	25:27:27	25:29:22	25:30:17	25:32:12	25:33:07	25:34:02	23:51:30	23:52:25	23:53:19	23:55:15	23:57:10	23:58:05
	Departure	25:27:36	25:29:31	25:30:27	25:32:22	25:33:17	25:34:12	23:51:40	23:52:35	23:53:29	23:55:25	23:57:19	23:58:15
N14	Arrival	25:47:45	25:48:45	25:50:45	25:51:45	25:53:45	25:54:44	23:58:45	23:59:45	24:01:45	24:02:45	24:04:45	24:05:45
	Departure	25:48:00	25:49:00	25:51:00	25:52:00	25:54:00	25:55:00	23:59:00	24:00:00	24:02:00	24:03:00	24:05:00	24:06:00

Coincidence			
20	20	15	16
15	20	15	20
20	14	15	20
15	20	15	10
20	20	20	20
15	20	19	16
5	15	20	15
20	16	20	15
15	17	16	15
15	20	16	20
14	20	13	16
20	19	14	20
20	20	13	20
15	20	15	20
15	20	20	15
20	17	20	20
19	17	20	20
19	20	20	8
19	20	13	20
15	16	19	15
10	20	10	20
20	15	16	10
20	20	13	16

me represents each coincidence (some colours are repeated, so the first time they occur represent the overlapping time of the first time they appear in the previous table).

In the following table, the time differences of initial and final timetables are presented. The intensity of the colour is related with the overlapping time. The main schedule changes are in the middle of the table and need to be anticipated several stations in advance.

The cumulative distribution function of the overlapping time for the second case is depicted in Figure 2. The high frequency of the maximum value (20 seconds) means that there is room to increase this limit (established by the current processes) and therefore to increment the total coincidence time. Many overlapping times correspond to intervals greater than 10 seconds.

5. Conclusions

Table 2

Time differences between timetables (optimal timetable calculated optimizing only departure times)

Differences		Stations																																																					
Trains	Events	PC	VA1	TE1	E1	AL1	CC1	RR1	I1	B1	T1	JA1	S1	TM1	AM1	AT1	AR1	MP1	P1	V1	NN1	PO1	BA1	AA1	MH1	SG1	VV1	COM	CO	VV2	SG2	MH2	AA2	BA2	PO2	NN2	V2	P2	MP2	AR2	AT2	AM2	TM2	S2	JA2	T2	B2	I2	RR2	CC2	AL2	E2	TE2	VA2	PCM
N1	Arrival			-5	-11	-15	-20	-25	-30	-35	-40	-45	-50	-55	-60	-65	-70	-75	-81	-56	-62	-67	-71	-71	-76	-50	-26				25	20	25	20	16	11	10	5		-5	-10	-15	-5	-10	15	10	5			-5	-10	-16	-20	-25	
	Departure		-5	-10	-16	-20	-25	-30	-35	-40	-45	-50	-55	-60	-65	-70	-75	-80	-56	-61	-66	-71	-70	-75	-50	-25				25	20	25	20	15	11	10	5		-5	-10	-15	-5	-10	15	10	5			-5	-10	-15	-20	-25		
N2	Arrival			-5	-10	-15	-20	-25	-30	-35	-40	-45	-50	-56	-60	-66	-71	-52	-53	-28	-32	-37	-42	-47	-51	-38	-25				-5	-10	-15	-5	15	40	35	30	25	20	15	10	5		-5	-10	-16	-20	-25	-30	-35	-40	-45	-20	
	Departure		-5	-10	-15	-20	-25	-30	-35	-40	-45	-50	-55	-60	-65	-70	-52	-52	-27	-32	-37	-42	-47	-51	-38	-25				-5	-10	-15	-5	15	40	35	30	25	20	15	10	5		-5	-10	-15	-20	-26	-31	-36	-40	-45	-21		
N3	Arrival			-5	-10	-15	-20	-25	-31	-35	-41	-17	-21	-27	-32	-36	-41	-41	-46	-26	-31	-37	-41	-35	-41	-45	-25				-5	-10	-15	-5	-10	-15	-20	-25	-29	-4	21	16	11	6	1	-4	-9	-14	-19	-24	-29	-34	-39	-14	
	Departure		-5	-10	-15	-20	-25	-30	-35	-40	-16	-21	-26	-31	-36	-41	-41	-46	-26	-31	-36	-42	-35	-40	-46	-25				-5	-10	-15	-5	-10	-15	-20	-25	-30	-4	21	15	10	6	1	-5	-10	-15	-19	-24	-29	-34	-39	-14		
N4	Arrival			-6	-10	-16	7	4	-2	-7	-11	-16	-21	-26	-31	-36	-41	-41	-46	-31	-35	-35	-10	-15	-19	6	1				-5	-10	-15	-20	-25	-30	-34	-39	-22	3	28	23	18	13	8	3	-2	-7	-12	-17	-22	-27	-32	-7	
	Departure		-5	-10	-15	8	4		-6	-11	-16	-21	-26	-31	-36	-41	-41	-46	-30	-36	-35	-10	-15	-19	6	1				-5	-11	-15	-21	-26	-30	-35	-39	-22	2	27	23	18	13	8	3	-2	-7	-12	-17	-22	-27	-32	-7		
N5	Arrival			-6	-11	-11	-16	-5	20	24	19	14	10	10	4		-5	-10	-15	-20	-25	-30	-35	-40	-45	-50	-25				-5	-10	-15	-20	-25	-30	-35	-40	-45	-47	-22	-15	-20	-25	-30	-35	-41	-45	-50	-55	-50	-55	-50	-25	
	Departure		-5	-10	-11	-16	-5	20	24	19	15	9	10	5	-1	-5	-10	-15	-20	-25	-30	-35	-40	-45	-50	-25				-6	-10	-15	-20	-25	-30	-35	-40	-45	-47	-22	-15	-20	-25	-30	-35	-40	-45	-51	-56	-51	-55	-50	-26		
N6	Arrival			20	15	10	14	14	9	3		-5	-10	-15	-20	-25	-25	-30	-20	5		-5	-10	-15	-15	10	5				-5	-10	-15	-20	-25	-30	-35	-40	-44	-30	-5	-9	-14	-19	-24	-29	-34	-39	-44	-49	-54	-59	-35	-25	
	Departure		20	15	10	15	13	9	4	-1	-5	-10	-15	-20	-25	-25	-30	-20	5		-5	-10	-15	-15	10	5				-5	-10	-15	-20	-25	-30	-35	-40	-45	-30	-5	-10	-15	-19	-24	-30	-35	-40	-44	-49	-54	-59	-35	-25		
N7	Arrival			11	5		-4	-9	-10	-15	-20	-25	-30	-35	-40	-45	-35	-10	-15	-20	-25	-30	-35	-40	-45	-50	-25				-5	-10	-15	-20	-25	-30	-35	-40	-45	-50	-30	-35	-40	-45	-25	-30	-11	-17	-21	-26	-32	-36	-41	-25	
	Departure		11	6		-4	-9	-10	-15	-20	-25	-31	-35	-40	-45	-35	-10	-15	-20	-25	-30	-35	-40	-45	-50	-25				-5	-10	-15	-20	-26	-30	-35	-40	-45	-50	-30	-35	-40	-45	-25	-30	-11	-16	-21	-26	-31	-36	-41	-24		
N8	Arrival			-5	-10	-15	-20	-25	-30	-25	-30	-35	-10	-15	-20	-25	-30	-10	15	10	5		-5	-10	-15	10	5				-5	-10	-15	-20	-25	-30	-35	-35	-40	-45	-32	-36	-42	-47	-51	-57	-36	-41	-30	-5	-11	-15	-20	-25	
	Departure		-5	-10	-15	-20	-26	-30	-25	-30	-35	-10	-15	-20	-25	-30	-10	15	10	5		-5	-10	-16	9	4				-5	-10	-15	-20	-25	-30	-35	-35	-40	-45	-31	-36	-41	-46	-51	-56	-36	-41	-29	-6	-10	-16	-20	-25		
N9	Arrival			4		25	25	25	20	15	9	5		-5	-10	-15	-20	-25		-5	8	3	-2	-7	-12	10	5				-5	-10	-15	-20	-26	-30	-31	-37	-42	-27	-3	-8	-11	-17	5		-5	-10	-12	-17	-21	-25	-30	-25	
	Departure		4		25	25	25	20	15	10	5		-5	-11	-16	-20	-26		-6	8	3	-3	-8	-12	10	5				-5	-10	-15	-20	-25	-30	-31	-36	-41	-27	-2	-7	-11	-16	4		-6	-10	-12	-17	-20	-26	-30	-25		
N10	Arrival			-5	-10	-14	-19		25	20	16	12	7	2	-3	-8	-13	-18		-5	-10	-15	-20	-25	-15	10	5				-5	-10	-15	-20	-25	-26	-30	-35	-10	-15	2	-3	-8	-13	-18	-23	-28	-25	-30	-35	-40	-45	-50	-25	
	Departure		-5	-11	-15	-19		24	19	16	11	6	1	-3	-8	-13	-18		-5	-10	-15	-20	-25	-15	10	5				-5	-11	-15	-21	-26	-26	-31	-35	-10	-16	1	-3	-8	-13	-18	-23	-28	-25	-30	-35	-40	-45	-50	-25		
N11	Arrival			-5	-10	-9	-14	-19	-24	-27	-32	-37	-42	-47	-52	-50	-50	-25		-5	-10	-15	-20	-25	-15	10	5				25	25	20	15	17	12	7	2	-3	5	30	25	29	24	19	14	9	4		-6	-11	-16	-21	-25	
	Departure		-6	-10	-9	-14	-19	-24	-27	-32	-37	-42	-47	-52	-50	-50	-25		-5	-10	-16	-20	-25	-16	9	4				25	25	20	15	17	12	7	2	-3	5	30	25	29	24	18	14	9	3	-2	-7	-11	-16	-22	-26		
N12	Arrival			-5	-10	-2	-7	-12	-5	-10	-16	-20	-25	-30	-35	-40	-45	-50	-55	-60	-65	-70	-75	-75	-50	-25	-25				-6	-7	18	14	14	8	3	-1	-6	-11		-6	-11	-17	-20	-25	-26	-30	-34	-35	-40	-40	-45	-20	
	Departure		-5	-10	-2	-7	-12	-5	-10	-15	-20	-25	-30	-36	-41	-45	-51	-56	-61	-65	-70	-76	-76	-50	-25	-25				-5	-6	19	14	14	9	4		-6	-11		-6	-11	-16	-21	-25	-26	-30	-34	-36	-40	-40	-45	-20		
N13	Arrival			16	41	36	31	26	51	52	47	42	37	32	27	22	17	12	7	2	-3	-8	-13	-18	-23	-20	-25				-6	-10	-10	-16	-16	-22		-5	-11	-15	-20	-26	-30	-35	-40	-45	-50	-50	-51	-56	-50	-55	-50	-25	
	Departure		16	40	35	31	26	50	51	47	41	36	31	27	22	17	12	7	2	-3	-8	-13	-18	-23	-20	-25				-5	-10	-10	-15	-17	-21	-1	-5	-10	-16	-20	-25	-31	-35	-41	-45	-50	-50	-51	-56	-50	-55	-50	-25		
N14	Arrival																																																						
	Departure																																																						

Table 3

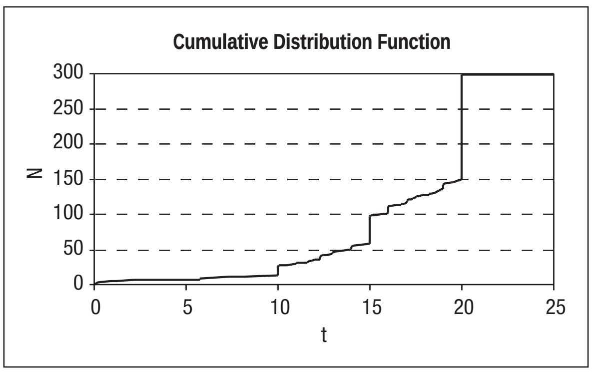
Time differences between timetables (optimal timetable calculated optimizing arrival and departure times)

Differences		Stations																																																					
Trains	Events	PC	VA1	TE1	E1	AL1	CC1	RR1	I1	B1	T1	JA1	S1	TM1	AM1	AT1	AR1	MP1	P1	V1	NN1	PO1	BA1	AA1	MH1	SG1	VV1	COM	CO	VV2	SG2	MH2	AA2	BA2	PO2	NN2	V2	P2	MP2	AR2	AT2	AM2	TM2	S2	JA2	T2	B2	I2	RR2	CC2	AL2	E2	TE2	VA2	PCM
N1	Arrival			-5	-11	-15	-20	-25	-30	-35	-40	-45	-50	-55	-60	-65	-70	-75	-81	-56	-62	-67	-71	-71	-76	-50	-26				25	29	24	19	15	10	5		-5	-10	-15	-20	-25	-30	-10	-15	-20	-5		-5	-10	-16	-20	-25	
	Departure		-5	-10	-16	-20	-25	-30	-35	-40	-45	-50	-55	-60	-65	-70	-75	-80	-56	-61	-66	-71	-70	-75	-50	-25				25	29	24	19	14	10	5		-5	-10	-15	-20	-25	-30	-10	-15	-20	-5		-5	-10	-15	-20	-25		
N2	Arrival			-5	-10	-15	-20	-25	-30	-35	-40	-45	-50	-56	-60	-66	-71	-52	-53	-28	-32	-37	-42	-47	-51	-29	-25				-5	-10	-15	-5	-10	-15	-20	-20	-25	-30	-15	10	5		-5	-10	-16	-20	-25	-30	-35	-40	-45	-25	
	Departure		-5	-10	-15	-20	-25	-30	-35	-40	-45	-50	-55	-60	-65	-70	-52	-52	-27	-32	-37	-42	-47	-51	-29	-25				-5	-10	-15	-5	-10	-15	-20	-20	-25	-30	-15	10	5		-5	-10	-15	-20	-26	-31	-36	-40	-45	-26		
N3	Arrival			-5	-10	-15	-20	-25	-31	-35	-41	-17	-21	-27	-32	-36	-41	-41	-46	-26	-31	-37	-41	-25	-31	-35	-25				-5	-10	-15	5		-5	-10	-15	-19	-5	20	16	11	6	1	-4	-9	-14	-19	-24	-29	-34	-39	-14	
	Departure		-5	-10	-15	-20	-25	-30	-35	-40	-16	-21	-26	-31	-36	-41	-41	-46	-26	-31	-36	-42	-25	-30	-36	-25				-5	-10	-15	5		-5	-10	-15	-20	-5	20	15	10	6	1	-5	-10	-15	-19	-24	-29	-34	-39	-14		
N4	Arrival			-6	-10	-16	7	4	-2	-7	-11	-16	-21	-26	-31	-36	-41	-41	-46	-31	-35	-35	-11	-16	-20	5					-5	-10	-15	-20	-25	-30	-35	-40	-22	3	28	23	18	13	8	3	-2	-7	-12	-17	-22	-27	-32	-7	
	Departure		-5	-10	-15	8	4		-6	-11	-16	-21	-26	-31	-36	-41	-41	-46	-30	-36	-35	-11	-16	-20	5					-5	-11	-15	-21	-26	-30	-36	-40	-22	2	27	23	18	13	8	3	-2	-7	-12	-17	-22	-27	-32	-7		
N5	Arrival			-6	-11	-11	-16	8	20	24	19	14	10	10	4		-5	-10	-15	-20	-25	-30	-35	-40	-45	-50	-25				-5	-10	-15	-20	-25	-30	-35	-40	-45	-47	-22	-15	-20	-25	-30	-35	-41	-45	-50	-55	-46	-51	-50	-25	
	Departure		-5	-10	-11	-16	8	20	24	19	15	9	10	5	-1	-5	-10	-15	-20	-25	-30	-35	-40	-45	-50	-25				-6	-10	-15	-20	-25	-30	-35	-40	-45	-47	-22	-15	-20	-25	-30	-35	-40	-45	-51	-56	-47	-51	-50	-26		
N6	Arrival			-5	-10	10	14	14	9	3		-5	-10	-15	-20	-25	-30	-35	-40	-45	-50	-55	-60	-35	-15	10	5				-5	-10	-15	-20	-25	-30	-35	-40	-41	-30	-5	-9	-14	-19	-24	-29	-34	-39	-44	-49	-54	-59	-34	-25	
	Departure		-5	-10	10	15	13	9	4	-1	-5	-10	-15	-20	-25	-30	-35	-40	-45	-50	-55	-60	-35	-15	10	5				-5	-10	-15	-20	-25	-30	-35	-40	-42	-30	-5	-10	-15	-19	-24	-30	-35	-40	-44	-49	-54	-59	-34	-25		
N7	Arrival			11	5		-4	-9	-5	-10	-15	-20	-25		5	5	5		-5	-10	-15	-20	-25	-30	-35	-40	-25				-5	-10	-15	-20	-25	-30	-35	-40	-45	-50	-30	-35	-40	-45	-25	-30	-11	-17	-21	-26	-32	-36	-41	-25	
	Departure		11	6		-4	-9	-5	-10	-15	-20	-26		5	5	5		-5	-10	-15	-20	-25	-30	-35	-40	-25				-5	-10	-15	-20	-26	-30	-35	-40	-45	-50	-30	-35	-40	-45	-25	-30	-11	-16	-21	-26	-31	-36	-41	-24		
N8	Arrival			-5	-10	-15	-20	-24	-29	-25	-30	-35	-35	-40	-45	-20	-25		25	20	15	10	5		-5	10	5				-5	-10	-15	-20	-25	-30	-35	-35	-40	-45	-32	-36	-42	-47	-51	-57	-36	-41	-17	-5	-11	-15	-20	-25	
	Departure		-5	-10	-15	-20	-25	-29	-25	-30	-35	-35	-40	-45	-20	-25		25	20	15	9	5		-6	9	4				-5	-10	-15	-20	-25	-30	-35	-35	-40	-45	-31	-36	-41	-46	-51	-56	-36	-41	-16	-6	-10	-16	-20	-25		
N9	Arrival			4		25	20	25	20	15	9	5		-5	-10	-15	-20	-25		-6	8	3	-2	-7	-12	10	5				-5	-10	-15	-20	-26	-26	-31	-37	-42	-27	-3	-8	-11	-17	5		-5	-10	-12	-16	-21	-25	-30	-25	
	Departure		4		25	20	25	20	15	10	5		-5	-11	-16	-20	-26	-2	-7	8	3	-3	-8	-12	10	5				-5	-10	-15	-20	-25	-26	-31	-36	-41	-27	-2	-7	-11	-16	4		-6	-10	-12	-16	-20	-26	-30	-25		
N10	Arrival			-4	-9	-14	-19		25	20	16	12	7	2	-3	-8	-13	-18		-5	-10	-15	-20	-25	-15	10	5				-5	-10	-15	-20	-25	-26	-31	-35	-40	-23	2	-3	-8	-13	-18	-23	-28	-25	-30	-35	-40	-45	-50	-25	
	Departure		-4	-10	-15	-19		24	19	16	11	6	1	-3	-8	-13	-18		-5	-10	-15	-20	-25	-15	10	5				-5	-11	-15	-21	-26	-26	-32	-35	-40	-24	1	-3	-8	-13	-18	-23	-28	-25	-30	-35	-40	-45	-50	-25		
N11	Arrival			-5	-10	-9	-14	-19	-24	-27	-32	-37	-42	-47	-52	-50	-50	-25		-5	-10	-15	-20	-25	-15	10	5				25	25	20	15	17	12	7	2	-3	22	35	34	29	24	19	14	9	4		-6	-11	-16	-20	5	
	Departure		-6	-10	-9	-14	-19	-24	-27	-32	-37	-42	-47	-52	-50	-50	-25		-5	-10	-16	-20	-25	-16	9	4				25	25	20	15	17	12	7	2	-3	22	35	34	29	24	18	14	9	3	-2	-7	-11	-16	-21	4		
N12	Arrival			-5	-10	-2	-7	-12	-5	-10	-16	-20	-25	-30	-35	-40	-45	-50	-55	-60	-65	-70	-75	-75	-50	-25	-25				-2	-7	18	14	14	8	3	-1	-6	-11		-6	-11	-17	-20	-25	-26	-30	-34	-30	-35	-40	-45	-20	
	Departure		-5	-10	-2	-7	-12	-5	-10	-15	-20	-25	-30	-36	-41	-45	-51	-56	-61	-65	-70	-76	-76	-50	-25	-25					-6	19	14	14	9	4		-6	-11		-6	-11	-16	-21	-25	-26	-30	-34	-31	-35	-40	-45	-20		
N13	Arrival			20	45	40	35	30	27	52	47	42	37	32	27	22	17	12	7	2	-3	-8	-13	-18	-23	-28	-3				-6	-10	-10	-16	-16	-22		-5	-11	-15	-20	-26	-30	-35	-40	-45	-50	-45	-50	-55	-50	-55	-50	-25	
	Departure		20	44	39	35	30	26	51	47	41	36	31	27	22	17	12	7	2	-3	-8	-13	-18	-23	-28	-3				-5	-10	-10	-15	-17	-21	-1	-5	-10	-16	-20	-25	-31	-35	-41	-45	-50	-45	-50	-55	-50	-55	-50	-25		
N14	Arrival																																																						
	Departure																																																							

The model presented in this paper is a decision support tool that can be used for maximizing the overlapping time between the slow-down and speed-up processes of underground trains in order to achieve energy savings. This problem has been formulated as a MIP optimization problem with a combinatorial nature and very difficult to solve. However, quasioptimal solutions can be obtained in a reasonable amount of time and show the dramatic potential savings achievable.

Figure 2

Cumulative distribution function of overlapping time



As observed in the case study most of the overlapping time is achieved without modifying the current advertised timetable and, therefore, with no public knowledge.

This model can be easily extended to consider several underground lines supplied by some coincident electrical substations or more trains in the same line (night and early bird trains, for example).

With this formulation it can be possible to count overlaps among multiple trains (more than two). Multiple overlaps have been manually accounted for and represent a 0.5 % of the total overlapping time estimated by the model for this case study, mainly due to the scarcity of the night trains. However, a future extension of the model has to include this possibility.

6. Acknowledgements

The authors of the paper acknowledge the collaboration of Metro de Madrid in providing data and following the model development.

7. References

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