Integrated offshore wind farm design: Optimizing micro-siting and cable layout simultaneously

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Funding information
US National Science Foundation, WINDINSPIRE, Grant/Award Number: OISE 1243482

Abstract
Electrical layout and turbine placement are key design decisions in offshore wind farm projects. Increased turbine spacing minimizes the energy losses caused by wake interactions between turbines but requires costlier cables with higher rates of failure. Simultaneous micro-siting and electrical layout optimization are required to realize all possible savings. The problem is complex, because electrical layout optimization is a combinatorial problem and the computational fluid-dynamics calculations to approximate wake effects are impossible to integrate into classical optimization. This means that state-of-the-art methods do not generally consider simultaneous optimization and resort to approximations instead.

We extend an existing model that successfully optimizes cable design to simultaneously consider micro-siting. We use Jensen’s equations to approximate the wake effect in an efficient manner, calibrating it with years of mast data. The wake effects are precalculated and introduced into the optimization problem. We solve simultaneously for turbine spacing and cable layout, exploiting the tradeoffs between these wind farm features. We use the Barrow Offshore Wind Farm as a case study to demonstrate realizable savings up to 6 MEUR over the lifetime of the plant, although it is possible that unforeseen design constraints have implications for whether the savings seen in our model are fully realizable in the real world. In addition, the model provides insights on the effects of turbine spacing that can be used to simplify the design process or to support negotiations for surface concession at the earlier stages of a project.

KEYWORDS
cable layout, economics, offshore wind, turbine micro-siting, wake modeling

1 INTRODUCTION

The Paris Agreement, currently ratified by 142 countries, aims to limit global warming over the next hundred years to 1.5°.1,2 In order to meet these aggressive targets, the European Union must reduce carbon by 20% by 2020 and by 80% by 2050.2 In Europe, offshore wind farms have a promising future, especially in the Irish, Baltic, and North seas where shallow waters provide inexpensive access to consistent, substantial wind resources.3 These changes are economically feasible and have substantial financial support.2,4 In 2016 alone, 10.3 GW of new European wind power capacity was financed.5 While there are still significant technological and economic barriers, it is feasible for the United States to achieve 20% or more of its energy from wind supply.6 As onshore space becomes scarce, offshore wind farm technology has become essential to meet these targets in both Europe and the United States.7

Decisions about the location of offshore farms (macro-siting),8-10 specific turbine placement in the layout (micro-siting),10-12 and electrical design10 are all problems faced by offshore wind farm development. Electrical design is closely tied to micro-siting and accounts for approximately...
20% of project costs,\textsuperscript{13} so it is a relevant consideration in wind farm layout design. In addition, offshore wind farms have higher repair times and costs than their onshore counterparts, so failure rate modeling is also an important consideration when determining layouts.\textsuperscript{10,14,15}

This paper investigates the benefits that can be achieved by integrating turbine placement optimization concurrently with cable layout design while considering wake effects. The contributions of this paper are the following:

- It proposes a method to calibrate the wake-effect description of the model in an efficient manner, based on clustering years of mast data.
- It proposes a method to consider simultaneously both effects. This required careful integration of multiple layout possibilities in the constraints, including a new methodology to capture variability in turbine power output between layouts.
- It presents the extension to a previous model called OWL (Offshore Wind Farm Layout Model)\textsuperscript{10} for simultaneously optimizing micro-siting and cable layout.
- It carries out a real case study based on Barrow Offshore Wind Farm (BOWF), which demonstrates the high potential savings associated with concurrent optimization.
- It provides insight on the tradeoffs associated with turbine spacing and wake effects. This information can be used to simplify the design process or to support permit negotiations at the earlier stages of a project.

This article is organized as follows. First, a brief review of existing offshore wind farm design models is presented in Section 1. Then, candidate modeling methodologies are discussed, with a focus on wake effects (Sections 2 and 3). The developed model and case study are then described (Sections 4 and 5). Finally, results are presented and conclusions offered (Sections 6 and 7).

2 LITERATURE REVIEW

Here, we summarize the state of the art for addressing the two problems that are simultaneously solved by the developed model: turbine placement (known as micro-siting) and cable layout. Cable layout costs can represent approximately 20% of the costs of wind farm installation and can vary between layout choices by approximately 10%. Savings associated with the wake effect vary by approximately 2% of the total cost of projects, giving it a similar magnitude impact as cable costs.

2.1 Turbine placement

Turbine placement optimization, also known as micro-siting, deals with the tradeoff between energy production and investment cost as turbine configurations are changed. At the most basic level, it accounts for the energy loss generated by the wake effect in comparison with the cost of spacing turbines further apart. Electrical component installation, surface concession, and environmental impact may all be relevant to this calculation.\textsuperscript{16} Existing micro-siting models usually focus on maximizing energy production while constraining to farm construction budgets, maximizing a definition of profit, or minimizing energy cost.\textsuperscript{17}

All fluid dynamics models found in the literature are based on Katic’s refinement of Jensen’s model as described in Katic et al\textsuperscript{18} because of its simplicity and accuracy. Models may\textsuperscript{19} or may not\textsuperscript{20} consider wind direction and wake effect variation.

Most methods for optimizing turbine placement consider expansions and contractions of standard layouts. Some models test several standard configurations and compare the optimal spacing for each configuration.\textsuperscript{21} Classical mixed-integer programming has been employed to solve this problem up to optimality.\textsuperscript{22,23} However, a large range of metaheuristic techniques have also been employed, including multiobjective evolutionary algorithms,\textsuperscript{16,24} gradient search,\textsuperscript{17,24} greedy heuristics,\textsuperscript{17,24} genetic algorithms,\textsuperscript{17,21,24-26} simulated annealing,\textsuperscript{17,20,24} particle swarm optimization,\textsuperscript{19} and pattern-search algorithms.\textsuperscript{17,24} Only a very reduced subset of these works considers unconventional layouts.\textsuperscript{20,27}

Electrical layout installation costs are crucial when considering variable turbine spacing.\textsuperscript{16} However, most models resort to simple heuristics for an approximation of cable layout costs, with the minimum-spanning tree being the most popular.\textsuperscript{16}

To the best of our knowledge, there are no previously published models that deal simultaneously with micro-siting and cable layout without resorting to simplifications or standard configurations. However, our model still lacks the generality of some heuristic turbine spacing models,\textsuperscript{20,27} because the model requires finitely many turbine placement layouts to be prespecified as input. Wake-effect costs are not well approximated linearly, so any mixed-integer linear programming (MILP) model must select which turbine configurations will be considered preoptimization so that wake costs can be precalculated for each layout.

2.2 Electrical layout optimization

The importance of the electrical layout in offshore wind farms has motivated the application of a wide array of techniques. A complete survey can be found in Lumbreras and Ramos.\textsuperscript{28} The electrical layout is composed of two parts: the collector system (which links the wind turbines) and the
transmission system (that takes the power to the point of common coupling in the onshore grid). The options considered for the collector system
are generally reduced to standard designs,\textsuperscript{10,13,28-30} such as stars, single-sided rings, double-sided rings, radial layouts, and multirings. However,
the optimal layout has a strong dependency on the precise layout of turbines, which greatly influences installation costs and failure rates. As a
result, standard configurations are rarely optimal.\textsuperscript{10,31} Only a few works allow for flexible designs, but this is done at the expense of using heuristic
techniques for layout design rather than classical optimization.\textsuperscript{10,32,33}

Reliability is a very important factor in layout design, as repairs offshore are difficult and costly. Failures can be approximated deterministi-
cally,\textsuperscript{34,35} modelled as scenarios in a stochastic program\textsuperscript{31,36} or simulated.\textsuperscript{37,38} However, most models ignore their effects.\textsuperscript{39,40}

The transmission system is responsible for sending generated power to the point of common coupling with the electrical grid. Several options
exist for this transmission\textsuperscript{11}:

- MVAC, for small amounts of power being transmitted short distances.
- HVAC, which elevates voltage using a transformer.\textsuperscript{11,40} As volume of power and distance to shore increase, so do the losses. This is currently
  the most common solution\textsuperscript{42} but is expected to become less common if farm sizes continue to increase and move away from shore.
- HVDC, which enables more efficient transmission of large amounts of power over greater distances.\textsuperscript{29,42,43} It also allows for connection to
  weaker grids.\textsuperscript{40} HVDC transmission is considered in a few existing models.\textsuperscript{12,41}

Different modelling compromises can be chosen with respect to power flow calculations. Transportation modeling,\textsuperscript{31,36} DCLF,\textsuperscript{35,45} and
ACLF\textsuperscript{40,46} methods for calculating power flows can all be used. In all models, losses can be ignored\textsuperscript{45} or approximated by linear or quadratic
functions.\textsuperscript{36,47} ACLF implementations usually approximate losses.

The cable layout problem can be solved as a classical MILP by representing losses using linear approximations,\textsuperscript{10} and decomposition strategies
can be used for computational savings.\textsuperscript{10,31,48} Nonclassical strategies such as heuristics,\textsuperscript{49} genetic algorithms,\textsuperscript{47} or immune system algorithms\textsuperscript{47}
have also been applied in this setting.

The tool of this paper is designed to refine micro-siting and cable layouts and not to address other important but broader questions, such as the opti-
mal turbine size and optimal number of turbines to place in a given area. When making these decisions at an earlier design stage, heuristic optimization
algorithms like particle swarm or genetic algorithms might produce general approximate results in a computationally efficient manner.\textsuperscript{19,25-27,32,35,47} If
different turbine layouts were suggested by different algorithms and for different numbers of turbines, then it would be appropriate to use our more
computationally expensive classical optimization tool to very precisely determine and compare the optimal arrangement for each layout suggested.

3 | METHODS: INTEGRATING ELECTRICAL LAYOUT AND TURBINE PLACEMENT

The model presented in this paper is an MILP that allows for the simultaneous optimization of turbine placement and cable layout, considering
flexible configurations and stochastic failures, resorting to classical optimization to guarantee global optimality (see Section 4). The model builds
on the model developed in Lumbreras and Ramos,\textsuperscript{10} which optimized cable layout taking turbine placement as an input. The objective function
weighs investment cost against the cost of energy lost due to wake effects and cable failures. This calculation is based on relative turbine and
substation coordinates for each farm, as well as years of mast data for the site under consideration.

Wake effects were introduced using Jensen's model as applied to entire wind farms in Peña et al.\textsuperscript{50} The model is equipped to consider complex wind
rose data from any number of directions at any number of velocities. The model automatically determines which wind scenarios to consider (the scenario-
tree centroids) and how to weight (probability) each wind scenario by using the K-means algorithm.\textsuperscript{51} These simplified data are then used to determine
the wake cost component of the objective function for each turbine placement option being considered. This turbine placement selection is introduced
as a new variable in the layout optimization problem. In order for objective functions corresponding to different layouts to be comparable, layout-specific

![Flowchart](https://example.com/flowchart.png)

**FIGURE 1** Flowchart of the three general stages of the model's operation. First, a pre-solve algorithm approximates turbine power outputs for
each layout considered under representative wind scenarios. Then, the core MILP model is run using the results from the first stage. Finally,
onimal layouts and the components of their objective functions are analyzed to inform the continuous distance selection problem of a given
turbine layout scheme
Wake-related factors were carefully integrated into calculations such as power not served. An alternative approach is to use brute force enumeration, wake simulation, and optimization of electrical layouts of every possible farm configuration. However, this approach is computationally unmanageable when there are thousands or millions of possible farm designs. Our optimization model implicitly considers all those layouts in a highly efficient manner and relatively quickly identifies optimal farm configurations and electrical layouts, considering wind variability and wakes.

The reliability of these layouts is considered using scenarios in which each component fails based on calculations of failure and repair rates assuming a discrete Markov process. Multiple components failing simultaneously are not considered by the model. The model is flexible enough to account for surface concession costs associated with each turbine layout if necessary (Figure 1).

4 | WAKE MODELING

Wake effects modify the power output produced at each turbine. In order to take them into account, the model takes years of mast data on the scale of minutes and turns them into a representative wind rose that can be used for the more intensive Jensen model calculations for each turbine.

Micro-siting options are modeled through a set of discrete variables that define the spacing among turbine rows and the distance between two consecutive turbines in a row. For each spacing option, the relative distances between turbines are calculated. Then, the model creates a large number of wind-speed and wind-direction bins. Mast data are then used to calculate the appropriate wind speed and direction values for each bin associated with the specific placement considered. For each wind speed bin, the weights for the corresponding wind direction bins are aggregated. Jensen's model is run on all wind direction bins, so that each turbine has an approximated power output for each wind direction bin. The power outputs for each turbine are then combined in a weighted sum to approximate power output for each turbine for each wind direction and speed bin. For a turbine under a given wind speed and wind direction scenario, Jensen's model is calculated as described in the following paragraphs.

The local speed deficit for a turbine caused by the wake of another turbine is approximated as in Peña et al. Let $\delta$ be local speed deficit, $C_t$ is the thrust coefficient, $r_r$ is the rotor radius, $k_w$ is the wake decay coefficient, and the turbines are at a distance $x$ from each other.

$$\delta = \frac{1 - \sqrt{1 - C_t}}{1 + k_w x/r_r}.$$

The total wind speed deficit for a turbine based on the deficit coefficients imposed on it by the turbines in whose wake it falls is then calculated as in Peña et al. using the quadratic sum of the square of local speed deficits. Let the turbine under consideration be in the wake of $n$ other turbines:

$$\delta_{\text{total}} = \left( \sum_{i=1}^{n} \delta_i \right)^{\frac{1}{2}}.$$

The incoming wind velocity is then calculated for each turbine as in Peña et al. Let $u_{\text{farm}}$ be the incoming wind velocity to the wind farm and $u_{\text{local}}$ be the incoming wind velocity to the turbine.

$$u_{\text{local}} = u_{\text{farm}} (1 - \delta_{\text{total}}).$$

The power output for the turbine is then approximated through a linear extrapolation of the two wind-velocity-to-power data points that its velocity falls between. Due to the high number of wind speed bins necessary for precise turbine power calculations, the bins must be condensed into wind speed scenarios. We use K-means clustering to condense these scenarios. The K-means algorithm then clusters wind speed bins with respect to the weight of each bin using the following objective function:

$$kmeansobj = \sum_{i=1}^{n} d_i \times \min_j(|p_i - m_j|),$$

where $n$ is the number of wind speed bins, $d_i$ is the weight of wind speed bin $i$, $p_i$ is the sum of turbine output power over all turbines in all configurations for wind speed bin $i$, and $m_j$ is mean $j$.

After the K-means algorithm converges, wind speed bins are grouped into scenarios based on the centroid they fall closest to. Power outputs for each turbine for each wind speed scenario are approximated by taking a weighted sum of the power outputs for each turbine over all wind speed bins present in the scenario. Let $B$ be the set of all wind speed bins included in the scenario, $p_b$ be the power output of the turbine for wind speed bin $b$, $d_b$ be the weight of wind speed bin $b$, and $p_{\text{scenario}}$ be the calculated power output of the turbine in the scenario. For a specific turbine in each wind speed scenario:
The duration of each wind speed scenario is calculated by taking the sum of the weights of the wind speed bins present in the scenario multiplied by the number of hours in a year. Let \( \alpha \) be the duration of a given scenario.

\[
\alpha = \left( \sum_{b \in B} d_b \right) \times 8.76.
\]

An ideal turbine power output calculation is also made for each wind speed scenario by calculating the power output for the turbines under that scenario if they were infinitely spaced. Note that all turbines would have the same power output if they were spaced far enough that the wake effects were negligible so long as incoming winds were roughly equivalent everywhere on the wind farm. The energy loss due to the wake effect of a given layout must be considered in two parts of the problem. First, for each layout under consideration, a cost is calculated to account for energy not served by the plant compared with the ideal scenario where turbines experience no wake effect. Second, power not served due to failure is based on power output for each turbine calculated after the wake effect is incorporated.

5 | MILP MODEL FORMULATION

The extended OWL model considers multiple input turbine layout proposals and returns the optimal turbine layout or micro-siting scheme. In addition, it returns the optimal collector and transmission systems for that offshore wind farm. Turbine layouts are given, together with a prespecified point of common coupling (PCC) and possible locations for offshore substations. Which cables are to be considered for installation, as well as available cable types, transformers and converters are also specified as inputs. The problem formulation below shows the full formulation of the MILP problem considering both micro-siting and cable layout. We list the indices, parameters, decision variables, constraints, and the joint objective function.

1. Indices:
   a. Configuration of the wind farm
      - \( p, p' \): geographical points where elements can be placed
      - \( \text{wt}(p), \text{cp}(p), \text{ps}(p) \): specific geographical points for the turbines, the point of common coupling and the offshore substations
      - \( d \): turbine layouts under consideration (e.g., row separation of 1000, 1050, 1100, or 1150 m)
   b. Equipment
      - \( c_t, c_{tac}(ct), c_{tdc}(ct) \): types of cable considered, subset of AC, and subset of DC types
      - \( v_l, v_{ldc}(vl), v_{lw}(vl) \): voltages that can be used, followed by a DC subset and the voltage level of turbines
      - \( t_t \): type of transformer or converter
      - \( \text{vs} \): side of the voltage (upper or lower). This set is used for enforcing voltage consistency.
      - \( r \): parallel-element index. If several elements are installed in parallel, having a different value for this index allows to differentiate them.
   c. Stochasticity
      - \( w_s \): scenario for wind input
      - \( s_s \): scenario for component failure.

In the following sections, a subindex refers to component failures, while a superindex denotes wind input.

2. Parameters:
   a. Geometry
      - \( D_{pp'} \): distance between two points (m)
   b. Components
      - \( CP, C_{Ct}, C_{Xt}, C_{Rt}, C_{Lt}, C_{mt} \): capacity, investment cost, reactance, resistance, rate of failure and repair (MW, MEUR per km, pu, pu, failures per km per year and repairs per year, respectively). The binaries \( BCV_{ct,v} \) summarize the information on what voltage level corresponds to each cable type.
      - \( TP, T_{Ct}, T_{Vl}, T_{HV}, T_{lt}, T_{lt}, T_{mt} \): capacity, investment cost, voltage level (lower), voltage level (upper), coefficient for losses, rate of failure and repair (MW, MEUR, kV, kV, pu, failures per year and repairs per year, respectively). The binaries \( BTV_{tt,vs,vl} \) summarize what voltage levels correspond to a transformer or converter type.
3. Variables

a. Design variables, all binary
   - $v_d$: micro-siting layout choice
   - $v_{WTP}^{p}$: use wind turbine power outputs for selected distance
   - $v_{\phi}$: voltage level chosen for a point in the design
   - $o_{s}$: placement of a substation at a point
   - $c_{p, r, d}$: installation of a cable
   - $t_{f, r, v}$: installation of a transformer or converter station

b. Variables describing operation, all continuous:
   - $f_{p,s}$: power flow (MW)
   - $\theta_{p,s}$: voltage angle according to Kirchhoff’s Second Law (rad)
   - $w_{tp}^{s}$: curtailment for a turbine (MW)
   - $p_{p,s}$: energy deficit with respect to the available power (MW)
   - $p_{p, c, p}$: power delivered to point of common coupling (MW)
   - $l_{p, o}$: losses in a line (MW)
   - $l_{o, t, f}$: losses in a transformer (MW)

4. Constraints

The model enforces the following constraints:

a. Design constraints
   - Only one turbine configuration is selected by the tool:
     \[ \sum_{d} v_{d} = 1. \]  

b. Cables installed must correspond to the turbine configuration selected:
   \[ c_{p, r, d} \leq v_{d}, \quad \forall p, r, d. \]  

C. The turbine power outputs must correspond to the turbine configuration selected:
   \[ v_{WTP}^{p} = \sum_{d} v_{d} v_{WTP}^{p,d}, \quad \forall W, p \in wt(p). \]  

- There can be no cables connecting to substations that are not installed:
  \[ c_{p, r, d} \leq o_{s}, \forall p \in wt(p), p \notin cp(p). \]
• Same as above, enforced for transformers and converters:

\[
0_{p} \geq \sum_{i=1}^{n} t_{p,i} t_{r,i} t_{s,i} \geq \sum_{i=1}^{n} t_{p,i} t_{r,i} t_{s,i}.
\]  

(5)

• There can only be one transformer or converter type, although there can be elements in parallel:

\[
\sum_{i=1}^{n} t_{p,i} t_{r,i} t_{s,i} \leq 1, \quad \forall p, p', \quad \forall s
\]

(6)

• Two points can only be connected by a single cable type, although elements in parallel are permitted:

\[
\sum_{i=1}^{n} c_{p,i} c_{r,i} c_{t,i} \leq 1.
\]

(7)

• Artificial constraint to impose that parallel elements are installed in ascending order:

\[
c_{p,p'} c_{r,d} c_{t,d} = c_{p,p'} c_{r,d} c_{t,d} \quad \forall t, r, r' \quad \forall p, p'.
\]

(8)

• All nodes have a voltage:

\[
\sum_{p} v_{p,d} = 1.
\]

(9)

• Only the transformers that are compatible with the voltage of the node can be installed:

\[
\sum_{t,t', v_t, v_{t', v_s}} t_{p,t} t_{t', v_s} \leq 1 - v_{p,d}.
\]

(10)

• Same as above, expressed for cables:

\[
c_{p,p'} c_{r,d} c_{t,d} \leq \sum_{v_t, v_{t', v_s}} \left\{ v_{p,d} + \sum_{t, r, s} t_{p,t} t_{r,s} (BTV_{t,t', v_s} - BTV_{t,t', v_s}) \right\}
\]

(11)

• These constraints encourage connectivity. Each wind turbine must have a cable connected to it. The connection to shore must be able to carry the power output of the farm running at capacity. A transformer must be present before the onshore connection, and it must make sense in the context of the maximum power output of the plant. These constraints are not necessary but makes the formulation of the MILP problem tighter, reducing the feasible region without removing potentially optimal solutions:

\[
\sum_{v_t, v_{t', v_s}} t_{p,t} t_{t', v_s} \geq \text{card}(v_t) \text{WTP}
\]

(12)

b. Link constraints (which deal with design and operation)

• Summation of power not served:

\[
p_{\text{ns}} = \sum_{p, w_{\text{t}}} w_{\text{t}} p_{\text{ns}} \quad \forall v, w_{\text{s}}, s.
\]

(13)

• Capacity constraints. Flow between nodes is restricted by the capacity of the cable and the functionality of the cable in that scenario. Energy delivered to the onshore grid is limited by the installation and functionality of transformers:
Currently, generating between 91 and 546 possible elements to be modeled by the N \text{-} 1 failure criterion. Each layout considered was represented by

$$f_{pp'}^{\text{ws},ss} \leq \sum_{c,t,r} f_{p', p, c, t, r} \left(1 - \frac{\text{Far}_{pp',pp'}^{\text{ws},ss}}{\text{CP}_{ct}}\right) \cdot \text{CP}_{ct}$$

$$f_{pp'}^{\text{ws},ss} \geq -\sum_{c,t,r} f_{p', p, c, t, r} \left(1 - \frac{\text{Far}_{pp',pp'}^{\text{ws},ss}}{\text{CP}_{ct}}\right) \cdot \text{CP}_{ct}$$

(14)

$$f_{pp'}^{\text{ws},ss} - \text{WTP}, \text{card}(wt) \cdot \left(1 - \sum_{t,r} \text{tf}_{pp',pp'}^{t},r \right) \leq \sum_{t,r,ss} \text{tf}_{pp',pp'}^{t},r \cdot \text{Tp}, \left(1 - \text{Far}_{pp',pp'}^{t},r \right)$$

$$f_{pp'}^{\text{ws},ss} + \text{WTP}, \text{card}(wt) \cdot \left(1 - \sum_{t,r} \text{tf}_{pp',pp'}^{t},r \right) \geq -\sum_{t,r,ss} \text{tf}_{pp',pp'}^{t},r \cdot \text{Tp}, \left(1 - \text{Far}_{pp',pp'}^{t},r \right)$$

$$\forall p, p', \text{ ws, ss}.\text{ }$$

c. Operation constraints:

- Balance of energy at each node in each scenario (First Kirchhoff’s law):

$$\sum_{d} f_{d, p}^{\text{ws},ss} + \text{WTP}_{d, p, \text{powt}(p)} - \sum_{d} \text{loss}_{d, p}^{\text{ws},ss} = \text{wtp}_{d, p, \text{powt}(p)} + \text{ps}_{d, p}^{\text{ws},ss}$$

$$\forall p, \text{ ws, ss}.\text{ }$$

- Balance of voltage angles for AC cables (Second Kirchhoff’s law):

$$\sum_{p'} f_{pp', p}^{\text{ws},ss} \leq \left(\theta_{p}^{\text{ws},ss} - \theta_{p'}^{\text{ws},ss}\right) \frac{C_{X_{ct}}}{C_{X_{ct}} \cdot \text{ord}(r)} + M \left(1 - \sum_{r'} f_{pp', p, c, t, r} \right)$$

(15)

$$\sum_{p'} f_{pp', p}^{\text{ws},ss} \geq \left(\theta_{p}^{\text{ws},ss} - \theta_{p'}^{\text{ws},ss}\right) \frac{C_{X_{ct}}}{C_{X_{ct}} \cdot \text{ord}(r)} - M \left(1 - \sum_{r'} f_{pp', p, c, t, r} \right)$$

$$\forall p, p', c, t, r, d, \text{ ws, ss}.\text{ }$$

5. Objective Function

The total cost of the layout is minimized. The problem considers investment costs, the production deficit due to wake effects, losses and curtailment due to equipment failures. The cost of power not served and wake effect costs were both calculated in EUR unrealized profit per undelivered MWh that could have been produced. This is equivalent to maximizing total profit from selling energy but allowed for easier comparability of the objective function components. In particular, designs that deliver more power will be more profitable when these opportunity costs of unrealized power production are included in the objective in this manner:

$$\min \left\{ \sum_{d} f_{d, p}^{\text{ws},ss} \cdot \text{CPP}_{d, p, \text{powt}(p)} + \text{CPP}_{d, p, \text{powt}(p)} \cdot \text{wtp}_{d, p, \text{powt}(p)} + \text{ps}_{d, p}^{\text{ws},ss} \right\}$$

$$+ \text{CPP}\sum_{d} \text{maxturb}_{d}^{\text{ws},ss} \cdot \text{Dur}_{d}^{\text{ws},ss} \cdot \text{card}(wt) - \sum_{d, \text{powt}(p)} \text{Dur}_{d, p}^{\text{ws},ss} \cdot \text{WTP}_{d, p, \text{powt}(p)}$$

$$+ \text{CLoss}_{d, p, \text{ws, ws}}^{\text{ws},ss} \cdot \text{Dur}_{d, p, \text{ws}, ss}^{\text{ws},ss} \cdot \text{Prob}_{d, p, \text{ws}, ss}^{\text{ws},ss} \right\}$$

(17)

Although losses add considerable complexity, they have a limited impact on the layout.\textsuperscript{10} We incorporate them by means of a two-phase approximation.\textsuperscript{10} Sánchez-Martín et al\textsuperscript{53} discusses a more computationally intensive method for this calculation.

6 | CASE STUDY

The wind farm layout optimization model was applied to the Barrow Offshore Wind Farm, currently in operation, to demonstrate the potential savings compared with conventional wind farm design techniques. We chose to use Barrow as our case study because it is large enough to have significant and complex wake interactions. Barrow is an excellent example of a wind farm with a classical layout strategy that is still often used in the industry. Furthermore, it enables direct comparability with previous studies such as Lumbreras and Ramos.\textsuperscript{10} When commissioned by Centrica and Dong Energy in 2006, it was the largest offshore wind farm ever built. Barrow is located in the Irish sea, and it includes 30 Vestas V90-3MW turbines creating a 90 MW capacity wind farm.\textsuperscript{54,55} The turbines are evenly spaced in four rows, two with seven turbines, and two with eight. The voltage of electricity generated is modified by an offshore transformer before transmission to shore. A more detailed account of its components can be found in previous study.\textsuperscript{54}

Our model considers HVAC, MVAC, and HVDC transmission systems from two possible offshore substation locations. The collector system was also simultaneously optimized, unrestricted by classical collector patterns. Anywhere from one to six turbine layouts were considered concurrently, generating between 91 and 546 possible elements to be modeled by the N - 1 failure criterion.
its own decision variable. While the considered layouts were scaled versions of the actual Barrow Wind Farm Layout, the model is equipped to handle any input turbine layout.

The power curve for the V90-3MW was available for precise power generation estimates for incoming wind speeds approximated by Jensen’s model in each scenario. The turbine’s diameter is 90 m. The wind data used for the Jensen’s model calculations were derived from the 75-meter mast located at Shell Flats which was used for the original planning of the wind farm. We used 211 746 10-minute wind speed and direction data points, representing over a year and a half of data collection. When implementing Jensen’s model, the wake decay coefficient was taken from Peña et al. as its methodology proved to be suitable to approximate wake deficit (and thus energy production) in a simple manner.

The turbine’s thrust coefficient was approximated to be constant as in previous studies. The thrust coefficient was taken to be 0.78 based on models of the V80 turbine, which is similar to the 0.75 used in models of the Holec WPS-30. No thrust coefficient data were found for the V90. Even though the thrust coefficient diminishes for higher-wind velocities, the turbines operate near maximum capacity in this region despite the wake effect. While the thrust coefficient was taken to be constant in the case study, the model is equipped to use nonconstant approximations of the thrust coefficient.

The 10-minute wind speed data were split into 200 wind speed bins and 400 wind direction bins, running one Jensen simulation for each wind speed wind direction pair with corresponding mast data points. Wind direction bins are more numerous than wind speed bins because slight changes in wind direction can drastically change the turbine power outputs depending on whether a wake from one turbine hits another. The number of bins necessary was approximated by plotting average wind farm power outputs for an increasing number of bins until the outputs converged to within 1 MW of 32.5 MW. The 80 000 Jensen iterations were then sorted into 20 wind speed scenarios using 5000 K-means iterations. Multiple wind speed scenarios are necessary to ensure that the objective function is properly impacted if low cable power ratings lead to power not served under high-wind conditions.

The cost of power not served due to cable failures was set to be 80 EUR per megawatt hour, and the cost of power loss due to the wake effect was set to be 29.33 EUR per megawatt hour. Surface concession cost was set to be 0.059 EUR/m²/year, very similar to the 0.058 EUR/m²/year cost of the actual BOWF. Choosing the costs of power loss and surface concession required some effort. The literature indicates that a range of 10 to 80 EUR per megawatt hour is reasonable for the cost of power losses and a range of 0 to 0.2 EUR per square meter for the financial lifetime cost of surface concession depending on location, stage of planning, and government policy. Curves for other wake-loss costs were also produced to test the robustness of the solution.

The cost of power not served due to cable failures was set to be higher than the wake effect costs of power not served in order to be consistent with earlier work. The higher cost of power not served due to cable failures is mainly due to the time scale of the losses. Short-term losses due to failure can create additional costs due to breached energy contracts, short-term energy future positions, and loss of tax credits from renewable energy generation. Long-term power not sold caused by unrealized generation opportunities due to wakes only needs to consider expected market prices plus a forecast of any feed-in renewable premium. Also, the benefits of additional energy produced by investing in greater turbine spacing is diminished by the risks of taking on additional financial liabilities due to greater investment costs.

7 | RESULTS

7.1 | Wake model testing

To ensure that the wake model functions as expected, the wake model was run off-line for 16 orientations of the wind farm. Annual power output performance was calculated for a row spacing of 750 m and a turbine spacing of 500 m so that direct comparison could be made to the orientation actually implemented at Barrow. The results of the runs can be found in Table 1.

After the runs, it was verified that directions opposite each other correctly yielded the same power output because of the symmetry of the farm. Figure 2 also confirms that the power outputs reported in Table 1 make sense in the context of the considered wind rose. Wind produces the most power when flowing perpendicular to the farm, the direction referred to as the targeted wind direction in Figure 2. This is because the distance between rows in the Barrow layout is much greater than turbine spacing within rows. Directions with large wind flow in the targeted orientation (or 180° from that direction) experience the highest annual power output.

Interestingly, the best orientation is between NNW and N because there are frequent periods of NNW wind as well as SSE wind. SSW orientations also performed well, as predicted by the wind rose. While the SW orientation of the implemented Barrow layout was one of the best choices and aligned very well with the average wind direction, it had a power output roughly 1% lower than the optimal. When we ran Jensen’s model with all bin settings up to plus and minus 10% of the number of bins used by our model, power output approximations varied by no more than 0.5%. This means the 1% difference is likely to be significant and warrants further analysis of possible modifications of the layout orientation. The wide range of wind directions present at Shell Flats led to significant power output variation (~3%) among orientations. This reiterates the already known importance of multidirectional wind analysis when using Jensen’s model.
While 16 orientations were considered to test the wake model, only the implemented Barrow orientation was used for the full model implementation so that direct comparisons to the actual farm, as well as the previous OWL model, could be made. This also avoided the issue of adjusting offshore substation placement and the cost of onshore connection, which would have required further assumptions in our model.

### 7.2 Full model implementation

By comparing the model's predicted investment costs of the implemented layout to actual investment costs of the Barrow project, fixed investment costs across all layouts were approximated to be 11.82 MEUR per year. Most of these costs are related to turbine installation, turbine purchase, and administrative planning costs that are identical for all layout options. To approximate these fixed costs, we ran an objective function calculation on the actual implemented Barrow layout. We then subtracted off our approximated cable installation and substation costs for the implemented layout from the total investment costs incurred by Dong and Centrica during the construction of Barrow.

Given that no previous works had analyzed the impact of row distance and wake effect considering an optimal cable layout, we produced several curves to understand their dynamics. The results can be used to facilitate the design of large wind farms or, given that they provide insights on the energy benefits of using larger row distances, support surface concession negotiations at the earlier stages of a project.

The goal was to compare the modelled cost of the implemented layout built in 2006 to the optimal layout that maintains the same relative turbine configuration. The objective function approximates costs using data provided directly by the cable and turbine manufacturers and aligns with values in the literature. They should therefore be very similar to the ones used by the Barrow design team. As a consequence, the comparison of the implemented layout's objective function value to the optimal layout's cost is a reasonably accurate estimate of the cost savings (in real EUR of 2006).

### TABLE 1
Annual energy generation in kilowatt hour preloss based on wind farm orientation

<table>
<thead>
<tr>
<th>Wind Direction</th>
<th>Annual Power Output in Kilowatt Hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>N,S</td>
<td>289.3</td>
</tr>
<tr>
<td>NNE,SSW</td>
<td>282.6</td>
</tr>
<tr>
<td>NE,SW</td>
<td>287.1</td>
</tr>
<tr>
<td>NEE,SWW</td>
<td>287.3</td>
</tr>
<tr>
<td>E,W</td>
<td>285.7</td>
</tr>
<tr>
<td>SEE,NWW</td>
<td>286.3</td>
</tr>
<tr>
<td>SE,NW</td>
<td>281.3</td>
</tr>
<tr>
<td>SSE,NNW</td>
<td>289.5</td>
</tr>
</tbody>
</table>

Note. Rows are spaced 500 meters and the distance between rows is 750 m. Orientation is considered perpendicular to rows.

### FIGURE 2
The wind rose for Shell Flats, which was used for the original planning of Barrow, (left) is compared to the maximum possible annual power output for each orientation of the wind farm (right). The targeted wind direction is the direction from which wind flow will produce the highest power output. The black arrow indicates the orientation of the implemented farm, where rows of turbines 500 meters apart point NW, as in Figure 3.
Trials at 17 different scales of the Barrow layout were completed, such that at each configuration the turbine row spacing was 250 m greater than the spacing of turbines within the row. A curve with exponential and linear components (Figure 4) was fit to the optimal objective function values at each spacing regime, which was then used to calculate an optimal turbine row spacing of approximately 1260 m (Figure 3). A detailed comparison of the objective functions for the optimal and implemented layouts is presented in Table 2 and Figure 5. Investment cost and surface concession cost increased slightly in the optimal layout (0.16 and 0.10 MEUR per year respectively), while the wake cost decreased by 0.55 MEUR per year. The optimal electrical layout is slightly different from the implemented one, which is a simple radial setting. The differences in the two layouts are greatest close to the offshore substation, where two parallel transformers are used instead of a single transformer.

To demonstrate the computational advantages of integrating turbine placement alongside cable layout, four distances were run simultaneously (750 by 500, 900 by 650, 1050 by 800, and 1200 by 950) and compared with the computer time to run each distance separately. The integrated program was able to select the optimal distance and layout after only 3.7 days of computer time compared with the 4.6 days required to run each distance sequentially. This demonstrated a 20% computational time savings with only four distances. These savings are expected to be larger if more distance options were considered, as with many combinatorial problems. This will allow the consideration of a finer granularity in distance options.

Investment cost and wake cost across the optimal layouts were also fit to curves. The investment cost (Figure 6) had a linear fit ($R^2 = .9976$), and the wake cost (Figure 7) had an exponential fit ($R^2 = .9999$). This suggests that less computationally complex methods may suffice for determining optimal spacing of certain turbine layout regimes, especially for larger wind farms.

### Table 2

<table>
<thead>
<tr>
<th></th>
<th>Total Cost</th>
<th>Investment Cost</th>
<th>Wake Cost</th>
<th>Power Not Served Cost</th>
<th>Surface Concession Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Implemented Layout</td>
<td>3.08</td>
<td>1.40</td>
<td>1.18</td>
<td>0.39</td>
<td>0.11</td>
</tr>
<tr>
<td>Optimized Layout</td>
<td>2.78</td>
<td>1.56</td>
<td>0.63</td>
<td>0.38</td>
<td>0.21</td>
</tr>
</tbody>
</table>
The increases in investment cost should be approximately affine because most utilized connections have a linear increase in length and do not increase in capacity or redundancy in the range considered. If different layouts had different optimal electrical configurations, our model would correctly account for these changes because all reasonable cable connections between turbines are considered for each turbine spacing regime. The electrical layout was not predetermined. We hypothesize that the electrical configuration remains constant, because decreasing failure rates made possible by introducing thicker cables is not a cost-effective way to increase expected power served as the layout expands. Failures of even the thinnest and longest cables occur on average less than once every 60 years, so it is credible that slight increases in failure rates do not economically justify thicker, more expensive cable connections. The only other reason the layout would require thicker cables would be if significantly more power failed to be served as a result of the capacity of the cables. This does not occur because there are very few periods where the cables operate at capacity in any turbine placement scheme. Similarly, the cable connections selected between turbines remain the same as the layout is expanded because the configuration remains optimal in the range of turbine spacings considered.

The exponential fit of wake cost is most likely caused by the power curves of V90 turbines. Power not served varied more but could possibly fit an inverse quadratic ($R^2, .9844$). Although more work must be done to validate these models, they could be a useful tool to simplify the planning stage, which can be especially daunting in the case of larger wind farms.

We believe that most of the savings of our optimized layout could have been realized, although we recognize the limitations of modeling. To support the predictive power of our model, we confirm that it accurately reflects BOWF where, without calibration, it predicted 302.5 MWh annually compared with the actual production of 303.2 MWh annually. This is significantly more accurate than the 314.3 MWh annually forecast.
by the original BOWF project team. However, we recognize that factors beyond the scope of our model may have impacted our optimized layout. For instance, benefits of choosing larger turbine spacing would be available only if additional concession costs are not unusually high for the location of BOWF. Furthermore, other engineering considerations may have been a factor, such as space constraints or budget limitations unavailable to us that create significant discrepancies between our model world and the actual Barrow implementation. Regardless of whether all major factors relevant to the design of BOWF were accounted for, industry has indicated value in optimization models such as our own, as recently demonstrated by Orsted.

8 | CONCLUSIONS

Continued interest in offshore wind farm investment has led to an increased importance of taking full advantage of optimal wind farm layouts. While investment cost may be modelled linearly, the interaction of power not served under cable failure in conjunction with the wake effect is difficult to model. This calls for concurrent optimization rather than model approximations to account for electrical layout costs during turbine placement optimization.

The Extended OWL model presented in this paper optimizes turbine spacing and electrical layout decisions, such that the resulting wind farm layout described is optimal based on the turbine placement schemes considered. The model uses MILP to concurrently optimize turbine positioning and cable layout. It relies on Jensen’s approximation to deal with the wake effects, which are calibrated using years of mast data.

The model has been applied to Barrow Offshore Wind Farm, an existing wind farm, to assess its potential savings. The optimal solution found by Extended OWL improves the implemented layout by 300 000 EUR per year, or 6 MEUR over the life of the plant. This is a calculated 10% savings of combined power not served, surface concession, wake, and cable costs. The designs are very different with respect to turbine spacing despite maintaining the same alignment. The optimal solution found was in line with studies that dealt with optimal turbine spacing alone.

We cannot rule out the possibility that there were nonpublicly disclosed surface concession constraints at the planning stage of BOWF that led to a tighter-than-optimal design. Major design considerations beyond our model representation of Barrow, such as real-world power pricing and the sea bed’s impact on turbine installation costs, could potentially have impacted layout selection. This has implications for whether the savings seen in our model are fully realizable in the real world.

The model can also develop curves that represent objective function value as a function of layout scale for each alignment regime. These curves can be used to compare conventional and unconventional turbine placement schemes and assess design tradeoffs. These curves can also be used for surface concession negotiations or layout planning depending on the stage of the project. If surface concession costs have already been negotiated, the model can incorporate post-negotiation representations of surface concession costs associated with each turbine layout.

As in Lumbreras and Ramos, our model can be applied to much larger farms given adequate computational resources in order to precisely differentiate between heuristically calculated turbine placement schemes in the final stages of offshore wind farm optimization. Just as in the Barrow case study, we expect that larger farms would similarly benefit from integrating electrical layout and turbine placement optimization and that classical layout designs would favor greater distances between turbines if additional surface concession was available under similar wind conditions.
The resulting model is robust with respect to the main factors affecting the problem and can also directly compare layouts of completely different turbine placement schemes simultaneously. Such comprehensive modeling is essential to optimally account for tradeoffs that can be worth millions of euros in a single offshore project.

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