STOCHASTIC BIDDING IN ELECTRICITY SPOT MARKETS. A MIP-ORIENTED BENDERS DECOMPOSITION APPROACH

José M Fernández, Santiago Cerisola, Álvaro Baíllo, Andrés Ramos
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- Mathematical Programming Model
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- Numerical Application To The Spanish Electricity Market
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Introduction

• **Optimizing offer curves is still a challenge** for generation companies taking part in spot markets
  – A relevant part of their revenues stems directly from the spot market
  – Operation costs also depend on the results of the spot market
  – The spot market is a reference for longer-term transactions

• **Offer curves** derived with the optimization approaches proposed in the literature are in general **not valid** for real generation companies
  – They may not comply with the technical and strategic constraints required by the generation companies or with formal limitations imposed by the market operator
Introduction

• We present a **methodology** to optimize offer curves considering a more practical approach
  – We take **valid offer curves** as an **initial point** for the optimization
  – We introduce modifications in these offer curves in order to maximize the expected profit of the generation company while complying with the constraints imposed by the user

• This assumption has evident **advantages**:
  – Solution existence is guaranteed
  – Modifications suggested by the model may provide valuable insight for the generation company strategy
  – The resulting offer curves are valid to be submitted to the market operator
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• **Problem Description**
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Problem Description

• We consider a **generation company** that owns a mixture of generation technologies
  – The aim of the company is to **maximize** its **long-term profit** through the operation of the generation units

• We focus our attention on the **Spanish Day-Ahead Market**
Problem Description

- **Revenue** depends not only on the company’s offer curve but also on the offers submitted by **other agents**
  - We represent this effect by means of **residual demand curves**
  - Uncertainty will be considered by means of different **scenarios** of residual demand functions for each hourly auction
Problem Description

- Our method evaluates the impact of increasing/reducing the amount of energy offered in each block of the initial offer curves.
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Mathematical Programming Model

OBJECTIVE FUNCTION:

$$\max \text{ Profit } = \sum_n E[\text{Revenue}_n - \text{Cost}_n]$$

- Hourly piecewise-linear functions as approximations of residual demand functions and revenue functions

- Binary variables are needed to model these functions
Mathematical Programming Model

OBJECTIVE FUNCTION:

\[
\max \quad \text{Profit} = \sum_{n} E[\text{Revenue}_n - \text{Cost}_n]
\]

– Costs:
  - O&M and Fuel consumption for Thermal units
    \[
    c^t_n = o^t q^t_n + f^t \left( b^t u^t_n + a^t q^t_n / k^t \right)
    \]
  - Hydro and nuclear costs are neglected
Mathematical Programming Model

OBJECTIVE FUNCTION:

\[
\max \text{ Profit} = \sum_n E[\text{Revenue}_n - \text{Cost}_n]
\]

– Modelling uncertainty:
  • Different scenarios of residual demand curves (and their corresponding revenue functions).

Residual Demand Curves in hour 14

Residual Demand Curves in hour 21
Mathematical Programming Model

CONSTRAINTS:

• Modeling the **offer curves** submitted by the company:
  – Hourly **stepwise curves** consisting of blocks defined in energy and price.
  – **Modifications** are introduced in **existing** blocks.
Mathematical Programming Model

CONSTRAINTS:

• Modeling the Market clearing process:
  – **Two different situations** are considered
  – We explicitly consider the case of **partially accepted blocks**
  – Some **binary variables** are needed to model the Market clearing process
Mathematical Programming Model

CONSTRAINTS:

• Modelling generation units:
  – Modelling thermal units:
    • Operation Limits:
      \[ q^t_k = q_n^t \leq q^t_k = q_n^t \]
  – Modelling hydro units:
    • The total amount of energy offered at each price is forced to remain constant
      \[ \sum_n q_{n1} = 0 \]

• This constraint links the 24 hourly auctions
Mathematical Programming Model

MATRIX OF CONSTRAINTS:
Mathematical Programming Model

MATRIX OF CONSTRAINTS:
CONSTRANTS:

• We re-formulate this complicating constraint in order to create a stair-case matrix structure:
  – We introduce a new variable that represents the accumulated modifications in energy offered

\[
q_{n,i}^{acum} = q_{n-1,i}^{acum} + q_{n,i}^a \quad n = 1, 2, \ldots, 24
\]

– Now the previous constraint is formulated as:

\[
q_{24,i}^{acum} = 0
\]
Mathematical Programming Model

MATRIX OF CONSTRAINTS:
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Description of the Decomposition Algorithm

- **Benders** method is oriented to solve mathematical programming problems with a **L-Shape** structure
  - This structure permits identifying **two stages** in the problem that are known as first and second stage
  - **Variables** are usually identified as first-stage variables and second-stage variables
Description of the Decomposition Algorithm

- **Benders algorithm** iterates between the resolution of both stages.
  - First-stage is denoted the **master problem** and incorporates the part of the objective function corresponding to first stage variables and a partial approximation of the recourse function.

  \[
  \text{Master Problem}
  \]

  \[
  \begin{align*}
  \text{Max } cx + qy \\
  Tx + Wy = h
  \end{align*}
  \]

  \[
  \begin{align*}
  \text{Subproblem}
  \]

  \[
  \begin{align*}
  \theta(x) &= \max qy \\
  Wy &= h - Tx
  \end{align*}
  \]

  - The **recourse function** represents second-stage objective function value as a function of first-stage decisions.
Description of the Decomposition Algorithm

- In case of **multiple-stages problems**, the decomposition method is extended in a natural manner for problems with a stair case structure.
Description of the Decomposition Algorithm

• **Binary variables** complicates the construction of the recourse function approximation.
  – This recourse function is nor **convex** neither **continuous**.
  – Traditional Benders approximation needs to be revisited

• We follow the **Generalized Benders Decomposition** and approximates the recourse function by solving the **Lagrangian Relaxation** of the **subproblem**
  – The relaxed constraints are those that connect different stages
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Numerical Application To The Spanish Electricity Market

- **Fictitious** power generation company owning a number of randomly chosen generation units present in the **Spanish** power system

<table>
<thead>
<tr>
<th>TOTAL Nuclear</th>
<th>Hydro</th>
<th>Pumping</th>
<th>Fuel</th>
<th>Gas</th>
<th>Coal</th>
<th>CCGT</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>12.73</td>
<td>32.20</td>
<td>7.07</td>
<td>4.70</td>
<td>7.10</td>
<td>25.07</td>
</tr>
</tbody>
</table>

- **Initial offer curves** are constructed by aggregating the offers corresponding to the selected generation units in a certain **day of the past** (July 29th 2005)
Numerical Application To The Spanish Electricity Market

- A number of sets of **day-ahead market scenarios** is constructed selecting **different previous days** similar to the day of study.

![Residual Demand Curves in hour 14](image1)

![Residual Demand Curves in hour 21](image2)
Numerical Application To The Spanish Electricity Market

- Resulting optimization problems:

<table>
<thead>
<tr>
<th>Number of Scenarios</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Equations</td>
<td>18873</td>
<td>63082</td>
<td>125872</td>
<td>251692</td>
</tr>
<tr>
<td>Number of Variables</td>
<td>12215</td>
<td>37615</td>
<td>73961</td>
<td>146485</td>
</tr>
<tr>
<td>Number of Binary Variables</td>
<td>6211</td>
<td>18910</td>
<td>37874</td>
<td>75586</td>
</tr>
</tbody>
</table>

- **Direct Resolution Vs. Decomposed Resolution** (subproblems comprising 2 hourly auctions)

<table>
<thead>
<tr>
<th>Number of Scenarios</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct Solution Time</td>
<td>10 secs</td>
<td>8 h 30 min</td>
<td>&gt; 1 day</td>
<td>??</td>
</tr>
<tr>
<td>Decomposed Solution Time</td>
<td>2 min</td>
<td>20 min</td>
<td>40 min</td>
<td>2 h 30 min</td>
</tr>
</tbody>
</table>
Numerical Application To The Spanish Electricity Market

• Results submitting the **Original** Offer-curves:

<table>
<thead>
<tr>
<th>Total Profit [€]</th>
<th>4782377</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Accepted Quantity [GWh]</td>
<td>90.233</td>
</tr>
<tr>
<td>Weighted Average Price [€/MWh]</td>
<td>75.12</td>
</tr>
</tbody>
</table>

• **Results Optimizing a hydro unit** must-run energy:

<table>
<thead>
<tr>
<th>Total Profit [€]</th>
<th>4854244</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Accepted Quantity [GWh]</td>
<td>90.366</td>
</tr>
<tr>
<td>Weighted Average Price [€/MWh]</td>
<td>75.53</td>
</tr>
</tbody>
</table>

– Results after optimization show a **1.5% increase in profits** while not modifying significantly market clearing results
Numerical Application To The Spanish Electricity Market

• **Modifications** in Hydro Production:
Numerical Application To The Spanish Electricity Market

- **Market Clearing Results** comparison:

  ![Original Offer-curve](chart1)

  ![Modified Offer-curve](chart2)
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Conclusion

• **Slight modifications** introduced in the company’s original offer curves turn into an **increment** of company’s expected profit
  – This seems to **confirm** the **validity** of our approach.

• The use of **decomposition techniques** allow us to apply our model in a realistic manner, obtaining optimal offer curves that can be directly submitted to a **real electricity market**
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Mathematical Programming Model

CONSTRAINTS:

• Modeling the **Market clearing process**:
  – We explicitly consider the case of **partially accepted blocks**

\[
\begin{align*}
q & \quad p \\
\text{Quantity} & \quad \text{Price} \\
\text{ RPni} & \quad = \quad p \quad \text{Sni}
\end{align*}
\]
Mathematical Programming Model

CONSTRAINTS:

• Modeling the Market clearing process:
  – Some binary variables are needed to model the Market clearing process
Description of the Decomposition Algorithm

• **Infeasibilities** management
  – The implemented decomposition algorithm takes two phases
  – An **initial phase** in which the integrality requirements are removed and a solution of the LP relaxation is obtained by the use of the linear nested decomposition algorithm.
    • Infeasible decisions are avoided with the construction of a feasibility cut.
  – A **second phase** in which
    • Each stage subproblem is solved with **MIP techniques** in the forward pass
    • Each subproblem is solved with the **RL method** in a backward pass. In case of infeasibility it is solve the RL of the minimization of infeasibilities subproblem.
Numerical Application To The Spanish Electricity Market

• **Results** comparison:
Problem Description

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