Tight and Compact MILP Formulation of Start-Up and Shut-Down Ramping in Unit Commitment

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Tight and Compact MILP Formulation of Start-Up and Shut-Down Ramping in Unit Commitment

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Abstract—This paper presents a Mixed-Integer Linear Programming (MILP) formulation of Start-Up (SU) & Shut-Down (SD) power trajectories of thermal units. Multiple SU power-trajectories and costs are modeled according to how long the unit has been offline. The proposed formulation significantly reduces the computational burden in comparison with others commonly found in the literature. This is because the formulation is i) tighter, i.e. the relaxed solution is nearer to the optimal integer solution; and ii) more compact, i.e. it needs fewer constraints, variables and nonzero elements in the constraint matrix. For illustration, the self-Unit Commitment problem faced by a thermal unit is employed. We provide computational results comparing the proposed formulation with others found in the literature.

Index Terms—Mixed-integer linear programming, start-up & shut-down ramps, thermal units, unit commitment.

NOMENCLATURE

The main definitions and notation used are presented in this section for quick reference. Upper-case letters are used for denoting parameters and sets; and lower-case letters for variables and indexes.

A. Definitions

The following terminology is used in this paper to reference the different unit operation states, see Fig. 1.

- **online**: the unit is synchronized with the system.
- **offline**: the unit is not synchronized with the system.
- **up**: the unit is producing above its minimum output. During the up state, the unit output is controllable.
- **down**: the unit is producing below its minimum output, when offline, starting up or shutting down.

B. Indexes and Sets

| l ∈ L | Start-up type, running from 1 (hottest) to \( N_L \) (coldest). |
| t ∈ T | Hourly periods, running from 1 to \( N_T \) hours. |

C. Variables

| \( v_t \) | Start-Up in period \( t \) \([0, 1]\). Continuous variable which takes the value of 1 if the unit starts up in period \( t \) and 0 otherwise, see Fig. 1. |
| \( w_t \) | Shut-Down in period \( t \) \([0, 1]\). Continuous variable which takes the value of 1 if the unit shuts down in period \( t \) and 0 otherwise, see Fig. 1. |
| \( \delta_{l,t} \) | Start-Up type \( l \) in period \( t \) \([0, 1]\). Continuous variable which takes the value of 1 in the period where the unit starts up for the start-up type \( l \) and 0 otherwise. |

D. Parameters

- \( EP_t \): Forecasted price of energy in period \( t \) \([\$/MWh]\).
- \( C^{LV} \): Linear variable production cost \([\$/MWh]\).
- \( C^{NL} \): No-load cost \([\$/h]\).
- \( C^{SU} \): Start-up cost for the start-up type \( l \) \([\$]\).
- \( C^{SD} \): Shut-down cost \([\$]\).
- \( P_\ell^{SU} \): Power output at the beginning of the \( \ell \)th interval of the start-up ramp process \([\text{MW}]\), see Fig. 1.
- \( P_\ell^{SD} \): Power output at the beginning of the \( \ell \)th interval of the shut-down ramp process \([\text{MW}]\), see Fig. 1.
- \( P_\ell^{syn} \): Power output at which the unit is synchronized for start-up type \( l \) \([\text{MW}]\), \( P_\ell^{SU} = P_\ell^{syn} \), see Fig. 1.
- \( SPD \): Duration of the shut-down ramp process \([\text{h}]\).
- \( SU \): Duration of the start-up type \( l \) ramp process \([\text{h}]\).
- \( RD \): Maximum ramp-down rate \([\text{MW/h}]\).
- \( RU \): Maximum ramp-up rate \([\text{MW/h}]\).
- \( TD \): Minimum down time \([\text{h}]\).
- \( TU \): Minimum up time \([\text{h}]\).
- \( Ti^{SU} \): Minimum number of periods that the unit must be down for the start-up type \( l \) \([\text{h}]\).
not include these ramps because the resulting model will be considerably more complex, causing prohibitive solving times [4]–[6]. In addition, due to the increasing penetration of wind generation nowadays, thermal units are being shut down and started up more often [7]; therefore, a detailed modeling of the SU and SD processes in UC is required.

The application of direct Mixed-Integer Linear Programming (MILP) to solving UC is becoming increasingly popular due to improvements in MILP solvers. For example, PJM has switched from Lagrangian Relaxation (LR) to MILP to solve the UC-based problems [8]. LR was the dominant optimization technique for solving UC problems through problem decomposition, mainly because LR does not present a high memory requirement as does MILP. However, this problem is being overcome due to the breakthrough of MILP solvers. Currently, combination of pure algorithmic speedup and the progress in computing machinery has meant that solving MILPs has become around 100 million times faster over the last 20 years [9]. Furthermore, MILP provides significant advantages over LR such as the fact that (i) there is a proven global optimal solution and (ii) MILP models are easier to modify, which enhances modeling capabilities and adaptability, among other things [1], [10], [11].

The SU and SD ramps are explicitly modeled under the LR approach in [12] and under the MILP framework in [13] and [11]. In [12] and [13], only a single power trajectory for the SU process is modeled, while [11] considers different SU power trajectories depending on the unit’s prior down time. Furthermore, [11] proposes a complete self-UC formulation which takes into account different constraints (e.g. power reserves and quadratic production costs) and is adapted to the Greek market rules.

References [13] and [11] made the important contribution in proposing the first MILP formulations for single and multiple SU & SD ramps, respectively. However, their main drawback is the creation of large models which greatly increase the complexity of UC problems, thereby making them unattractive for practical implementation. These models are large due to the introduction of many constraints in order to deal with the power trajectories above and below the minimum output of generating units. Apart from this, [11] needs many binary variables to model the different SU power trajectories.

The use of MILP-based UC formulations has increased significantly over the last 50 years [14]. As computational and algorithmic power increases, so does the complexity of the MILP formulations, with the addition of features such as ramping constraints, minimum up and down times and exponential SU costs [2]. The computational burden of UC problems needs to be further reduced, by improving the MILP formulations, so that even more advanced and computationally demanding problems can be implemented, such as stochastic formulations [15], contingency-constrained models [16], and generation expansion planning [17].

Improving an MILP formulation allows a faster search for optimality by tightening (removing inefficient solutions from) the original feasible region. Tightening requires strong lower bounds for minimization problems [18]. This means formulating the problem in such a way that the associated linear programming (LP) relaxation provides a better approximation of the value of the integer optimal solution. The time required for providing optimality is often prohibitive because the gap between the integer optimal solution and its associated LP relaxation is very large. Furthermore, a poor lower bound provided by the LP relaxation will not be adequate to guide the search for good feasible solutions during the solving phase (branch-and-cut) of standard MILP solvers [19]. MILP formulations are frequently tightened by adding a huge number of constraints and (sometimes) variables. However, the resulting expanded model must close the gap enough to be worth the extra time taken to solve the LP relaxations during the solving phase [20]. In other words, usually, tightening an MILP formulation comes at the expense of expanding the model which implies extra time consumption. Therefore, creating tight and compact MILP formulations is a nontrivial task because the obvious formulations are commonly either very weak or very large.

Creating tight (or strong) MILP formulations has been widely researched [21]. In the case of UC problems, there has been work in a number of specific areas. In [22], a strong formulation of the minimum up/down time constraints is presented; in [23], a tighter linear approximation for quadratic generation costs is proposed; and [24] presents a new class of inequalities giving a tighter description of the feasible operating schedules for generators.

The main contribution of this paper is two-fold:

1) A tighter MILP formulation of SU & SD ramps for UC problems is proposed in order to reduce the computational burden of analogous existent MILP formulations.

2) This MILP formulation is also compact and hence overcoming the main disadvantage of previous models [11], [13]. If a single power trajectory is modeled for the SU & SD ramps, then there is neither a need to increase the number of constraints nor a necessity to increase the number of variables in comparison to a formulation without the SU & SD ramps. Furthermore, when considering different SU trajectories, the proposed formulation requires the introduction of merely continuous variables compared to [11]. Additionally, this formulation of SU & SD ramps is suitable for any UC problem, whether under centralized or competitive environments, and further model expansion will not require the introduction of numerous terms in the constraints in order to avoid conflicts between the up and down states, unlike [13] and [11].

In order to illustrate the effectiveness of the proposed formulation, the self-UC for a price-taker thermal generator is used. The objective of a thermal generator, in the self-UC, is to maximize the profits from selling energy in the day-ahead market, while satisfying all the technical constraints.

The reminder of this paper is organized as follows: Section II presents the formulation of the SU and SD constraints in detail. Section III provides and discusses results from several case studies, where the impact of neglecting the SU and SD ramps is shown and a comparison of the proposed formulation with those in [13] and [11] is made. Finally, some relevant conclusions are drawn in Section IV.
II. PROPOSED APPROACH

This section presents the mathematical formulation of the SU & SD power trajectories. With the purpose of illustrating how this formulation works, the objective function is formulated for the case of a price taker self-UC problem. This section is divided into two parts: Section II-A details the mathematical formulation and Section II-B shows how the optimization problem has been solved.

Hourly time intervals are considered, but it should be noted that the formulation can be easily adapted to handle shorter time periods. For the sake of simplicity, reserve constraints are not considered; however, they can be easily introduced in the model. The interested reader is referred to [5], [11], [25].

A. Mathematical Formulation

The different operation states of a thermal unit are presented in Fig. 1. The up and down states are distinguished from the online and offline states. During the up period, the unit has the flexibility to follow any trajectory being bounded between the maximum and minimum output and by the ramping-rate limits. On the other hand, the power output when the unit is starting up or shutting down follows a predefined power trajectory.

1) Up/Down vs. Online/Offline States: By considering the commitment variable $u_t$ as up/down rather than offline/online states, the generation output above and below $P$ can be managed independently. This characteristic makes the proposed formulation (i) compact, unlike [13] and [11], where most of the constraints involving $p_t$ contain summations of binary variables in order to avoid conflicts between the power output above and below $P$ and (ii) tight where, by considering the generation output ($p_t$) above $P$, the feasible region for $p_t$ is between $P$ and $P$, which is tighter than the region that is usually considered, between 0 and $P$.

The down times are a function of the offline times (see Fig. 1). For example, the number of periods that the unit must be down to activate the SU type $l$, $T_{SU}^{l}$, is equal to the SU & SD ramp durations ($SU^U$ and $SD^L$) plus the offline time required to activate the SU type $l$. Similarly, the down time $TD$ is expressed as a function of the minimum offline time (see Section II-A3).

2) Start-Up Type: Different SU types are modeled depending on how long the unit has been down. The SU type $\delta_{t,l}$ is selected if the unit has been previously down within the interval $[T_{SU}^{l}, T_{SU}^{l+1})$, see Fig. 2. Each SU type has a different SU power trajectory associated to it, where the colder the SU type, the longer the SU power trajectory duration (see example shown in Fig. 3 in Section III-A). As in the case of [26] and [11], new variables are introduced to select the SU type:

$$\delta_{t,l} \leq \sum_{i=0}^{T_{SU}^{l}-1} u_{t-i} \quad \forall t \in [T_{l+1}^{SU}, N_T], l \in [1, N_L] \ (1)$$

where the right side of (1) is equal to 1 if the unit has been down within the interval $[T_{SU}^{l}, T_{SU}^{l+1})$ before hour $t$. Therefore, $\delta_{t,l}$ can only be activated ($\delta_{t,l} \leq 1$) if the unit has previously been down within this interval.

Note that (1) is not defined for the first hours. Appendix A details how the first SU types $\delta_{t,1}$ are obtained depending on the initial conditions of the unit.

The following constraint ensures that just one SU type is selected when the unit starts up:

$$\sum_{l=1}^{N_L} \delta_{t,l} = v_t \quad \forall t \ (2)$$

Equation (1) constrains all SU types except the coldest one $\delta_{t,N_L}$. However, constraints (1) and (2) ensure $\delta_{t,N_L} = 1$ when the unit starts up ($v_t = 1$), and has been down for at least $T_{SU}^{l}$ hours. This is because (1) makes $\delta_{t,l} = 0$ for all $l \neq N_L$ and then (2) forces $\delta_{t,N_L} = 1$. In the event that more than one SU type variable can be activated ($\delta_{t,l} \leq 1$) then (2) together with the objective function ensure that the hottest, which is the cheapest, possible option is always selected. Therefore, just one of the variables is activated (equal to one). That is, these variables take binary values even though they are modeled as continuous variables. This is due to the convex (monotonically increasing) characteristic of the exponential SU costs of thermal units [2], see Fig. 2.

Constraint (1) is made even more compact by taking into account the minimum down time constraint (see Section II-A3). The hottest SU $\delta_{t,1}$ must be activated when the unit has been down within the interval $[0, T_{SU}^{1})$. However, the minimum down time constraint (4) ensures that the unit cannot be down for less than $TD$ hours. Therefore, the hottest SU is only possible within the interval $[TD, T_{SU}^{1})$. By defining...
$T_{SU}^1 = T_D$, see Fig. 2, constraint (1) together with the minimum down time constraint (4) ensure that the hottest SU $\delta_{i,1}$ can be activated only when the unit has been down less than $T_{SU}^2$ hours.

3) Minimum Up/Down Times: Constraints (3) and (4) ensure the minimum up and down times respectively [22]. This formulation has been compared with others and has shown a better performance [22], [24].

$$\begin{align*}
\sum_{i=t-TU+1}^{t} v_i & \leq u_t & \forall t \in [TU, N_T] \quad (3) \\
\sum_{i=t-TD+1}^{t} w_i & \leq 1 - u_t & \forall t \in [TD, N_T] \quad (4)
\end{align*}$$

The minimum down time $T_D$ in (4) is equal to (i) the SD ramp duration ($SD^D$), plus (ii) the hottest SU ramp duration ($SU^D_{i,1}$), plus (iii) the minimum time that the unit must be offline. Therefore, (4) is needed to avoid overlapping between the SU & SD ramps. Appendix A describes how the initial conditions force the unit to remain up/down during the first hours.

4) Commitment, Start-Up & Shut-Down: The following constraint can be found in models published approximately fifty years ago [14].

$$u_t - u_{t-1} = v_t - w_t & \forall t \quad (5)$$

Once $u_t$ is defined as a binary variable, (5) forces $v_t$ and $w_t$ to take binary values.

5) Capacity Limits: The generation level in UC problems is usually expressed as hourly energy blocks; however, it has been demonstrated that taking a generation level schedule as an energy delivery schedule may not be realizable [27], [28]. Therefore, a clear difference between power and energy is made and all technical constraints are then imposed over the power output variable. The power generation output of the unit above its minimum output is modeled as:

$$0 \leq P_t \leq \overline{P} - \underline{P} (u_t - w_{t+1}) & \forall t \quad (6)$$

Constraint (6) ensures that the total power output is equal to $\overline{P}$ ($P_t = 0$) at the beginning and at the end of a continuous up period. On the other hand, the SU (SD) trajectory ends (begins), during the down period, at $\underline{P}$ level, thereby making the connection with the up period that starts (ends) at this level, see Fig. 1.

6) Operating Ramp Constraints: As mentioned in Section II-A1, the proposed formulation avoids the introduction of many variables in most of the equations, unlike [13] and [11]. This is the case for the ramping constraints that only depend on the generation variables between two consecutive hours:

$$- RD \leq P_t - P_{t-1} \leq RU & \forall t \quad (7)$$

7) Energy Production: The total unit’s energy production, including the energy produced during the SU & SD processes, is presented in (8). Note that the energy is obtained for hourly periods and piecewise-linear power trajectories (see Fig. 1). However, the conversion to shorter time periods is straightforward.

$$e_t = \underline{P} u_t + \frac{P_t + P_{t-1}}{2} \sum_{i=1}^{SD^D} P_{i+1}^D + P_{i}^D u_{t-i+1}$$

$$+ \sum_{i=1}^{NL} \sum_{i=1}^{SU^D} \frac{P_{i+1}^SU + P_{i}^SU}{2} \delta_{t-i+SU^D_{i,1}} & \forall t \quad (8)$$

The terms of the summations in (8) include the energy produced during the SU & SD procedures.

Equation (8), together with (1) and (2), make a tight description of the SU & SD ramps in the energy output variable $e_t$. This could be observed from the fact that on the one hand, when the unit is starting up ($v_t = 1$), (1) and (2) will choose the correct SU type ($\delta_{t,1}, \frac{\delta_{t,1}}{2}$), and then the associated SU energy trajectory is immediately fixed in (8), while on the other hand, when the unit is not starting up ($v_t = 0$) then (2) forces all SU types to be zero ($\delta_{t,1} = 0$) and thus the SU energy in (8) is immediately fixed to zero. Similarly, the SD decision ($w_t$) will fix the SD energy trajectory in (8). Besides, the tightness of the formulation is experimentally checked in Section III, where the integrality gap of the proposed formulation is lower than those in [13] and [11].

Note that when just a single SU power trajectory is modeled, there is no need to introduce variables $\delta_{t,1}$. Therefore, constraints (1) and (2) are not needed and the scheduled energy in (8) must be modified to be directly affected by the SU variable $v_t$ instead of $\delta_{t,1}$.

8) Objective Function: The goal of a price-taker producer in a self-UC is to maximize his profit during the planning period, which is the difference between the revenue and the total operating cost (9). For the sake of simplicity, a linear production cost is used in this paper.

$$\max \sum_{t=1}^{N_T} \left[ EP_t e_t - \left( C^{NL} u_t + C^{LV} e_t + \sum_{l=1}^{C_{SU}^{NL}} C_{SU}^{NL} u_{t,l} + C_{SU}^{D} \delta_{t} + C_{SU}^{SD} w_{t} \right) \right] \quad (9)$$

Note that the no-load cost ($C^{NL}$) considered in (9) ignores the SU & SD periods. This is because the $C^{NL}$ only multiplies the commitment during the up state $u_t$. In order to consider the no-load cost during the SU & SD periods, $C_{SU}^{SU}$ and $C_{SD}^{SU}$ are introduced in (9) and defined as:

$$C_{SU}^{SU} = C_{SU}^{SU} + C_{SU}^{NL} u_{SU}^D \quad \forall i \quad (9a)$$

$$C_{SD}^{SU} = C_{SD}^{SU} + C_{SU}^{NL} u_{SD}^D \quad (9b)$$

B. Final Power Schedule

The complete energy profile, including SU & SD power trajectories, was presented in (8). Nevertheless, the complete power output as well as the unit states online/offline have not yet been obtained. This information can be explicitly modeled as variables in the optimization problem, which will create a considerably larger formulation. However, these values can be obtained after the optimization problem has been solved without changing the optimal results and then with negligible
computational cost. Furthermore, this also contributes to the compactness of the formulation. The total power output $P_t$ and online/offline states $U_t^{ON}$ are presented as follows:

$$P_t = P(u_t + v_{t+1}) + p_t + \sum_{i=2}^{SD+1} P_i^{SU} w_{t-i+2}$$

$$+ \sum_{l=1}^{N_L} \sum_{i=1}^{SU_D} P_i^{SU} \delta(t-(i+SU_D)+2),t \quad \forall t$$

$$U_t^{ON} = u_t + \sum_{l=1}^{N_L} \sum_{i=1}^{SU_D} \delta(t-(i+SU_D)+2),t + \sum_{i=1}^{SD} w_{t-i+1} \quad \forall t$$

Furthermore, analogously to the SU and SD decisions $v_t$ and $w_t$ which represent the changes between the up and down states, the turn-on $V_t^{ON}$ and turn-off $W_t^{OFF}$ decisions representing the changes between the online and offline states are now obtained with

$$V_t^{ON} = \sum_{l=1}^{N_L} \delta(t+SU_D),t \quad \forall t$$

$$W_t^{OFF} = w_{t-SD} \quad \forall t$$

### III. TEST RESULTS

The proposed formulation is tested for the self-UC of a price-taker producer. The technical and economic data for the thermal unit, including five different SU ramps, are presented in Table I, and the expected electricity prices for a 48-hour time span are shown in Appendix B. These data are based on information presented in [11]. The power outputs $P_t^{SU}$ ($P^{SD}$) for the SU (SD) power trajectories are obtained as an hourly linear change from $P_t^{SU}$ ($P^{SD}$) to $P_t^{SU}$ ($P^{SD}$) for a duration of $SU_D$ ($SD_D$) hours, see Fig. 1. With respect to initial conditions, the unit has been up for 6 hours before the scheduling horizon and its initial power output is 200 MW. All tests in this paper were carried out using CPLEX 12.3 under GAMS [29] on an Intel i7-2.4 GHz personal computer with 4 GB of RAM memory. Problems are solved to optimality, more precisely to 1e-6 of relative optimality tolerance.

This section is divided into two parts. The first part shows the impact of SU & SD ramps on the unit commitment. The second part presents a comparison of the proposed formulation with those presented in [13] and [11].

### Table I: Thermal Unit Data

<table>
<thead>
<tr>
<th>Technical Information</th>
<th>Cost Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$ [MW]</td>
<td>$L$ [MW]</td>
</tr>
<tr>
<td>378.0</td>
<td>150.0</td>
</tr>
</tbody>
</table>

### Start-Up Ramping Information

<table>
<thead>
<tr>
<th>SU Type</th>
<th>$C_{SU}$ [$/]</th>
<th>$P_{SU}^D$ [MW]</th>
<th>$SU_D$ [h]</th>
<th>$T_{SU}^D$ [h]</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>16</td>
<td>50</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>02</td>
<td>28</td>
<td>50</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>03</td>
<td>36</td>
<td>50</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>04</td>
<td>40</td>
<td>50</td>
<td>4</td>
<td>11</td>
</tr>
<tr>
<td>05</td>
<td>41</td>
<td>50</td>
<td>5</td>
<td>14</td>
</tr>
</tbody>
</table>

### Table II: Producer Costs and Profits

<table>
<thead>
<tr>
<th>Costs ($)</th>
<th>Revenues ($)</th>
<th>Profits ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional</td>
<td>576417</td>
<td>617252</td>
</tr>
<tr>
<td>Improved</td>
<td>402201</td>
<td>461674</td>
</tr>
<tr>
<td>% of change</td>
<td>-30.22</td>
<td>-25.20</td>
</tr>
</tbody>
</table>

Fig. 3: Optimal generation scheduling for the Traditional and Improved formulations in the up and bottom part of the figure respectively. The darker gray area shows the SU & SD energy that the unit will produce in order to follow the optimal schedule that results from the Traditional formulation (this extra energy is added after solving the Traditional problem).

A. Scheduling and Economic Impact

In order to illustrate how the unit operation is affected if the SU & SD ramps are considered, the case study has been solved with and without the ramp trajectories. The formulation with SU & SD ramps is labeled as Improved and the formulation considering just the exponential-SU & SD costs is labeled as Traditional.

Unlike the Traditional formulation, considering the energy produced during the SU & SD ramps makes the Improved formulation perceive revenues during these ramping processes. To make a fair comparison between both formulations, the inherent energy produced during the SU & SD ramps is introduced into the Traditional formulation after the problem has been solved (see darker gray area in Fig. 3). That is, even when the Traditional formulation ignores these ramps in the scheduling stage, they are inevitably present during the operation stage. Subsequently, this energy can be added to the solution and this extra energy can also be sold. The total revenues for the Traditional formulation are then obtained by adding the revenues obtained from the UC solution to the revenues obtained from the energy produced during the SU & SD processes. These latter revenues are calculated by multiplying the electricity price by the energy produced during the SU & SD ramps.

Fig. 3 and Appendix B show the optimal power and energy schedules for the Traditional and Improved formulations. Note the different duration of the SU power trajectories for the Improved formulation in Fig. 3. SU durations of one, two and three can be observed (starting at hours 16, 6 and 30 respectively) as a consequence of the different unit’s down
The optimal scheduling decision taken by the *Traditional* formulation around hours 15-17 and 39-41 was to produce at minimum output \( P_{\text{min}} \) (in Fig. 3) even when electricity prices were lower than the unit's linear variable production cost \( C_{\text{LV}} \) (CLV in Fig. 3). This is a very common behavior, where producing at \( P_{\text{min}} \) generates fewer losses than shutting down and starting up the unit within a short period. On the other hand, when SU & SD ramps are considered, the optimal scheduling decision is to turn off the unit during these hours. The reason is that the SU and SD costs are offset by the revenues received from the energy produced during the SU & SD ramping processes. In short, unlike the *Improved* formulation, in the *Traditional* model, the SU and SD processes are perceived as pure losses. Therefore, the optimal decision of the *Traditional UC* formulation is to not turn off the unit for short periods to avoid these losses.

The main problem affecting the *Traditional* formulation is that revenues during the SU and SD processes are not considered in the optimization problem. Therefore, there is a tendency to produce at least at minimum output, even when electricity prices are lower than \( C_{\text{LV}} \), and thus obtain some revenues that compensate for the losses. This drawback is overcome by considering the SU & SD power production in the formulation.

Table II shows the difference between costs, revenues and profits for the solutions of both formulations. As mentioned before, the total revenue for the *Traditional* model is obtained by adding the revenues due to the ramping process (\$14800) to the revenues obtained from the optimal solution (\$602452). For this illustrative case, the profits when considering the SU & SD ramps are around 46% higher than when these ramps are not taken into account.

### B. Comparing Different Formulations

The proposed formulation is compared with those available in [13] and [11]. Reference [13] proposes a formulation to deal with a single SU and SD ramp trajectory whereas [11] deals with different SU trajectories depending on the unit's prior down time. The different SU types, their associated costs and power-trajectories are inherent characteristics of thermal units and these data are provided by the manufacturer. However, in order to compare the different formulations, the proposed formulation is implemented considering one, three and five different SU ramp types, and these models are labeled as R1, R3 and R5 respectively. The single ramp type model R1 can be directly compared with the single-ramp formulation proposed in [13]. The model presented in [11] was implemented considering three SU ramp types, and hence it can be directly compared with R3. Model R5 is presented in order to observe the extra computational burden which results from considering extra SU ramp trajectories.

In order to assess the impact of the problem size on the computational performance of the models, several case studies of different sizes were solved. The price profile of one day (see Appendix B) has been replicated over different time spans from 4 to 256 days (each case is solved in one step for the complete time span). The unit data are presented in Table I, where the information for the single-ramp models (R1 and [13]) is the SU ramp type 02, the three-ramp models (R3 and [11]) are the first three SU types, and the five-ramp model (R5) are the five SU types.

1) **Assumptions for the Formulations:** In order to compare all the formulations, [13] and [11] were implemented using the same objective function and the same set of constraints as the formulation presented in Section II. Therefore, all the models are characterizing the same problem; the difference between them is how the constraints are formulated. In other words, two models considering the same SU types (R3 and [11], or R1 and [13]) obtain the same optimal results, e.g. commitments, generating outputs and profits.

The distinction between power and energy was made when implementing [13] and [11]. Additionally, as modeled here, the (usual) power variable is considered to be the power at the end of the period; and the energy is obtained by applying a piecewise-linear power profile. [11] was implemented with the same minimum up/down constraints presented in Section II-A3 as those are the constraints they also use. The synchronization time was set to zero in [11] as we believe this time does not need to be explicitly modeled, thus making the formulation simpler. That is to say, the synchronization time can be considered as a part of the offline time and obtained after the problem has been solved, without changing the optimal results (similar to the turn-on state presented in Section II-B). Finally, the other constraints presented in [11], which are not related to the SU & SD ramps, were not implemented (e.g. quadratic production costs and different power reserves).

Table III presents the optimal solutions for all the models for different time spans. As expected, the optimal solution for models R1 and [13] are the same, as well as the solution for models R3 and [11]. Interestingly, model R5, which considers five ramp types, also presents the same solution as R3 and [11]. This is because, in R5, ramp types 04 and 05 were never activated for this example case because the unit was not down for long enough. As in the case of the difference between R1 and R3 (see Table III), if the five different ramps had been activated in R5, this would have decreased the operational costs in comparison with R3 because more flexibility is possible (more SU ramp types).

2) **Problem Size:** Table III shows the dimension of all the models for the different case studies. Models R1 and [13] have the same number of variables, but [13] presents three times as many binary variables as R1. This is because [13] defines the SU & SD variables as binary; however, they can be considered as continuous variables (see Section II-A4). The formulation in [13] also requires more than twice the quantity of constraints and nonzero elements than the proposed formulation R1.

As shown in Table III, [11] presents about 16 times more binary variables than the proposed formulation R3. Models R3 and R5 have more real variables than [11]. However, the total number of variables in R3 and R5 is smaller than the number of binary variables in [11]. R3 and R5 also present less than half the number of constraints than [11]. Furthermore, [11] presents up to 4.3 and 3 times more nonzero elements than R3 and R5 respectively. Similarly to [13], [11] needs these extra
variables and nonzero elements to deal with the different SU ramp types and to avoid conflicts between the up and down states.

Note that model R5 is slightly larger than R3, with respect to the number of variables and constraints, because R5 considers two more ramp types than R3. This also shows that the compact formulation does not increase considerably when considering more SU types.

3) Computational Performance: Apart from the compactness of the proposed MILP formulation, the tightness has a significant impact on the computational performance, as mentioned in the Introduction. In fact, a compact formulation usually presents a weak LP relaxation that can dramatically increase the MILP resolution time. The tightness of an MILP can be measured with the integrality gap [24]. The integrality gap, for a maximization problem, is defined as $Z_{LP} - Z_{MILP}$, where $Z_{LP}$ is the optimal value of the relaxed LP problem, and $Z_{MILP}$ is the best integer solution found after the MILP problem is solved.

Table III shows the integrality gaps for the different formulations. Compared to [13], the proposed single-ramp formulation R1 has improved (reduced) the integrality gap between $46\%$ and $49\%$. Similarly, with respect to [11], R3 improves the integrality gap between $41\%$ and $45\%$. Table III also shows the nodes explored during the branch-and-cut phase; these are usually decreased with tighter formulations. Note that, for all the different cases, CPLEX was able to solve R1 with the required optimality tolerance without needing to branch, because the nodes were pruned earlier by the initial heuristics and cuts applied. Apart from the number of nodes, the performance of an MILP formulation is dramatically affected by the use of heuristics and cuts, and all of these are influenced by the tightness of the formulation [19]. Therefore, we will only comment about CPU times which offer a more complete view of the model’s performance.

The CPU times for the different case studies are presented in Table III, where R1 and R3 are up to 22.4 and 10.1 times faster than [13] and [11] respectively. This significant speed up is due to the simultaneous tightness and compactness of the proposed formulation. It is interesting to note that the formulation in [13], which models a single ramp trajectory and does not take into account exponential SU costs, is a larger model (presents more constraints and nonzero elements) and also requires more time to solve the problem than R5, which considers five different SU ramps and also exponential SU costs.

Finally, Fig. 4 shows the convergence evolution for the different formulations to small optimality tolerances for the case study of 256 days. The proposed formulation converges significantly faster than [13] and [11]. This is mainly due to its tightness.

IV. CONCLUSIONS

This paper presented an Mixed-Integer Linear Programming (MILP) formulation of the Start-Up (SU) and Shut-Down (SD) power trajectories of thermal units. This formulation is simultaneously tighter and more compact than equivalent formulations found in the literature. Consequently, the computation time is dramatically reduced. The proposed MILP formulation was analyzed in the context of a price taker self-unit commitment problem. However, its application to any unit commitment problem is straightforward, either under centralized or competitive environments. Several case studies
were analyzed to show the improvements of this formulation with respect to others available in the literature. Although SU & SD ramps are usually not considered, mainly because of the computation complexity, simulation results showed that ignoring them changes the commitment decisions causing a negative economic impact.

APPENDIX

A. Initial Conditions

The following parameters are needed to deal with the unit state during the first periods:

\[ u_0 \quad \text{Initial commitment state} \{0, 1\}. \]

\[ TU_0 \quad \text{Number of hours that the unit has been up before the scheduling horizon.} \]

\[ TD_0 \quad \text{Number of hours that the unit has been down before the scheduling horizon.} \]

1) Initial Minimum Up/Down Times: The following condition must be satisfied if \( (TU_R + TD_R) \geq 1 \):

\[ u_t = u_0 \quad \forall t \in [1, TU_R + TD_R] \] \hspace{1cm} (14)

where \( TU_R \) and \( TD_R \) are the number of initial hours during which the unit must remain up or down at the beginning of the scheduling horizon. \( TU_R \) and \( TD_R \) are defined as follows:

\[ TU_R = \max \{0, (TU - TU_0)u_0\} \] \hspace{1cm} (14a)

\[ TD_R = \max \{0, (TD - TD_0)1 - u_0\} \] \hspace{1cm} (14b)

2) Initial Start-Up Type: Equation (15a) complements (1) taking into account the initial conditions if \( TD_0 \geq 2 \):

\[ \delta_{t,l} = 0 \quad \forall l \in [1, NL], t \in (T_{I+1}^{SU} - TD_0, T_{I+1}^{SU}) \] \hspace{1cm} (15a)

Finally, the following equation guarantees that the SU \( \delta_{t,l} \) is not activated before the SU ramp duration \( SU^P \):

\[ \delta_{t,l} = 0 \quad \forall l, t \in [1, SU^P] \] \hspace{1cm} (15b)

In other words, this condition ensures that if the unit is turned on in the first hour, the power output above \( P \) is produced after the ramp duration \( SU^P \).

B. Generator-Schedules and Price-Data

The expected electricity prices and optimal power schedules mentioned in Section III are shown in Table IV, where super-indexes \( T \) and \( I \) refer to the Traditional and Improved formulations. The numbers in parentheses are the power and energy that were included after the UC problem was solved.

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German Morales-Españo has been awarded an Erasmus Mundus Ph.D. Fellowship. The authors would like to express their gratitude to all partner institutions within the programme as well as the European Commission for their support. The authors also would like to thank Prof. J. M. Arroyo for his valuable comments.

REFERENCES


Table IV: Price Data and Optimal Generation Schedule

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The authors introduce a tight and compact MILP formulation for start-up and shut-down ramping in unit commitment. The formulation is designed to improve upon existing MILP formulations by incorporating new constraints and inequalities. The objective is to optimize the operation of power systems, with a focus on start-up and shut-down ramping. The authors discuss the importance of such formulations in modern power system planning and operation, highlighting the benefits of using advanced optimization techniques to enhance system reliability and economic efficiency.

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