Goal Programming Approach to Maintenance Scheduling of Generating Units in Large Scale Power Systems

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Abstract—This paper presents a goal programming methodology for solving maintenance scheduling of thermal generating units under economic and reliability criteria. The advantages of a multicriteria approach will be demonstrated by comparing the effects that costs and reliability have on each other in power plants maintenance scheduling. The problem is formulated as a large scale mixed integer programming problem implemented in the mathematical programming language GAMS and solved using OSL. Weekly maintenance scheduling of the large scale Spanish power system for a year period illustrates the proposed methodology.

I. INTRODUCTION

Preventive maintenance of thermal generating units is a classical problem of resource scheduling in power systems. This is a large scale non linear stochastic optimization problem with constraints [3,8].

Firstly, the number of independent variables in a mathematical model is determined by the number of units (more than 100 for the very large systems) and by the number of time stages (frequently 52 weeks). This results in a very high number of possible solutions. The non-linearity is an inherent feature of power systems, manifested in the relationship between fuel consumption and generated power, limited energy units or the fact that many variables are integer. Finally, the stochastic nature of maintenance scheduling (MS) problem derives from load, thermal units forced outages and natural hydro inflows.

There exist several goals when solving planning and scheduling problems and, in most cases, they are antagonistic. However, the conventional approach in existing power systems optimization algorithms is such that the most important criterion is formulated as the optimization objective, while other criteria are defined as constraints or introduced into the objective function in the form of penalties. Such single-objective methods do not completely meet the requirements of utility planners, whose most important task is the determination of the best compromise solution of the objectives considered. The use of goal programming techniques is therefore a logical step in overcoming the lacks of single-objective models.

The maintenance scheduling model proposed is a medium term production cost model formulated as a large scale mixed integer optimization problem subject to operation and maintenance constraints. The optimization is performed in two stages including two different criteria: minimum total variable operation costs and minimum differences between the reserve margins of consecutive periods.

The paper is organized as follows. A short review of different techniques for solving the MS problem is presented in Section II. The proposed method and the model description are given in Sections III and IV. Its implementation is object of Section V. An application to the annual MS of the Spanish electric power system is described in Section VI. And finally the conclusions are extracted in Section VII.

II. SOLUTION APPROACHES

Several methods have been proposed in the literature to solve the maintenance scheduling of generating units. Different single-objective approaches were employed:

- Integer programming [4]
- Decomposition methods [14,16]
- Dynamic programming [15]
- Knowledge-based models [2,10]
- Simulated annealing method [12]
- Heuristic algorithms

The binary nature of the MS problem make a very reasonable method for integer optimization, e.g., the branch and bound approach can be applied in solving integer and mixed integer problems, as well as non linear integer problems. According to [8], it guarantees to find the feasible solution (if one exists) and the optimal solution with respect to the chosen optimality criterion. Also this method is computationally acceptable even for problems with a large number of variables and constraints.

All of those methods were used for solving the MS problem with only an objective criterion as costs (maintenance costs, variable operation costs), reliability (minimum sum of the reserve margin for each period, leveling the reserve margins for all the periods), etc.

But in MS a compromise between those requirements must be achieved: adequate reliability with minimal fuel
costs, maximum efficiency in using available resources or minimal difference between period reserve margins. These requirements are of conflicting nature; it is obvious that the MS problem is essentially of a multiobjective type. This was recognized by Mukerji et al. [9], discussing the solutions obtained by optimization of two alternative objective criteria: costs or reliability. But the first paper in proposing the goal programming approach to the MS problem, was done by Kralj, B., Rajakovic, N [7]. This paper was applied to a medium size system and only thermal operation is modeled. In this paper other resources such as pumped-hydro units, pumped-storage, fuel consumption with constraints are modeled and optimized simultaneously with MS within the proposed methodology. Furthermore, the model is applied to a large dimension power system.

III. PROPOSED METHOD

The proposed algorithm is shown in Fig. 1. This methodology is based on a sequential optimization process of both economic and reliability objectives. The economic purpose is the minimization of total variable operating costs (TOC1) (fuel + O&M + startup + unserved and interruptible energy). Other economic objectives have been proposed in the literature as maintenance costs, fixed and variable costs, maintenance crew costs, etc.

The second optimization run is done minimizing the sum of the differences between the thermal reserve margins of consecutive periods. The reserve margin is calculated dividing the available thermal capacity by the period peak load. Reliability has been defined in many ways such as loss of load probability (LOLP), loss of load expectation (LOLE), expected unsupplied energy (EUE), etc. Generally, it is shown that optimal solutions obtained under one reliability criterion are also acceptable in terms of the others [17]. The leveling reserve margins criterion obtains always less riskier solutions than those obtained by the optimization under other reliability criteria. Here, it has been used the net thermal reserve margins leveling between periods in a deterministic way: units availability is modeled derating the maximum unit power by its EFOR (equivalent forced outage rate), being a simplified way (but equivalent) to represent the random availability variable.

The constraints, as Fig. 1 shows, are those related with the system operation and with equipment maintenance. Second stage total operating costs, TOC2, are subject to the solution costs obtained for stage 1, multiplied by \( \beta \), a maximum percentage of TOC1.

\[
TOC_2 \leq TOC_1 \times (1 + \beta/100)
\]

\( \beta \) represents the compromise between both criteria, being the planner who chooses it adequately to the system and the solution desired.

IV. SYSTEM MODELING DESCRIPTION

This model proposed addresses all the operating decisions, adequately represented in two types: interperiod decisions are those regarding resources planning for multiple periods (in particular, maintenance scheduling for thermal units, seasonal operation of pumped-hydro units, and fuel scheduling are represented) and intraperiod decisions correspond to a economic generation dispatch (those related to daily operation of pumped-storage units and commitment decisions of thermal units).

![Graphical representation of the proposed method](image)

Fig. 1. Goal programming applied to the thermal MS problem.

Its prime purpose is to predict the future system operation and their parameters (costs, fuel consumption, productions, etc.) and determine the annual maintenance scheduling for every week and every thermal unit for the whole electric power system.

All the stochastic variables are treated in a deterministic way. The generators unavailability is modeled by derating the maximum capacity of power plants by its equivalent forced outage rate. Uncertainty in system load and in hydro
inflows are treated under the assumption of average demand and hydrology respectively.

Time scope of the model (duration of the time interval to be studied) is divided into periods, subperiods and load levels. Typically, periods (52) will correspond to weeks, subperiods (2) to weekdays and weekends, and load levels (3) to peak, plateau and off-peak hours. On the weekend subperiods only two load levels are considered. The load is modeled by a staircase representation.

The capacity of each thermal unit is divided into two blocks, being the first one the minimum load block. Heat rate is specified by a straight line with independent and linear terms. This model also allows a piecewise representation of the nonlinear consumption curve.

A. Notation

- Notations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$B$</td>
<td>number of pumped-storage units.</td>
</tr>
<tr>
<td>$C$</td>
<td>number of thermal plants with fuel quotas.</td>
</tr>
<tr>
<td>$C^*$</td>
<td>number of thermal plants.</td>
</tr>
<tr>
<td>$H$</td>
<td>number of hydro and/or pumped-hydro units.</td>
</tr>
<tr>
<td>$N$</td>
<td>number of load levels.</td>
</tr>
<tr>
<td>$P$</td>
<td>number of periods.</td>
</tr>
<tr>
<td>$S$</td>
<td>number of subperiods.</td>
</tr>
<tr>
<td>$T$</td>
<td>number of thermal units.</td>
</tr>
<tr>
<td>$A_{hp}$</td>
<td>hydro inflows expressed in energy for hydro unit $h$ in period $p$.</td>
</tr>
<tr>
<td>$b_p$, $b_b$</td>
<td>maximum and minimum capacity of pumped-storage unit $b$ when pumping.</td>
</tr>
<tr>
<td>$C_{cp}$</td>
<td>minimum quota of fuel consumption in thermal plant $c$ at the beginning of period $p$.</td>
</tr>
<tr>
<td>$c_c$</td>
<td>fuel storage cost in thermal plant $c$ per unit of time.</td>
</tr>
<tr>
<td>$d_{nsp}$, $d_{asp}$</td>
<td>customer and interruptible power demand in load level $n$ of subperiod $s$ of period $p$.</td>
</tr>
<tr>
<td>$D_{nsp}$</td>
<td>duration of load level $n$ of subperiod $s$ of period $p$.</td>
</tr>
<tr>
<td>$e_{hp}$, $e_b$</td>
<td>maximum and minimum capacity of pumped-hydro unit $h$ when pumping.</td>
</tr>
<tr>
<td>$g$</td>
<td>maximum number of thermal units simultaneously in maintenance on the same plant.</td>
</tr>
<tr>
<td>$h_{hp}$, $h_{bp}$</td>
<td>maximum and minimum capacity of hydro unit $h$ in period $p$.</td>
</tr>
<tr>
<td>$k_t$</td>
<td>auxiliary services coefficient of thermal unit $t$.</td>
</tr>
<tr>
<td>$m$</td>
<td>coefficient of the thermal installed capacity simultaneously on maintenance in any period.</td>
</tr>
<tr>
<td>$M_t$</td>
<td>number of periods on scheduled maintenance for the thermal unit $t$.</td>
</tr>
<tr>
<td>$o_p$, $o_t$</td>
<td>heat rate (independent and linear terms) of thermal unit $t$.</td>
</tr>
<tr>
<td>$P_p$, $P_b$</td>
<td>maximum and minimum rated capacity of thermal unit $t$.</td>
</tr>
<tr>
<td>$p_l^p$</td>
<td>peak load in period $p$.</td>
</tr>
<tr>
<td>$q_t$</td>
<td>EFOR of thermal unit $t$.</td>
</tr>
<tr>
<td>$R$</td>
<td>power reserve margin.</td>
</tr>
<tr>
<td>$R_h$, $R_b$</td>
<td>maximum and minimum hydro energy reserve of hydro unit $h$.</td>
</tr>
<tr>
<td>$r_f$</td>
<td>startup cost of thermal unit $t$.</td>
</tr>
<tr>
<td>$S_c$, $S_b$</td>
<td>maximum and minimum fuel storage capacity of thermal plant $c$.</td>
</tr>
<tr>
<td>$t_f$, $t_b$</td>
<td>maximum and minimum capacity of pumped-storage unit $b$ when generating.</td>
</tr>
<tr>
<td>$w_t$</td>
<td>O&amp;M variable cost of thermal unit $t$.</td>
</tr>
</tbody>
</table>
| $V_{bh}$ | upper reservoir limit of pumped-storage unit $b$.
| $v_{fr}$ | fuel cost of thermal unit $t$. |
| $W$ | penalty cost by power reserve defect. |
| $w_b$, $w_p$ | unserved and interruptible energy cost. |
| $\eta_b$ | performance of pumped-storage unit $b$. |
| $\eta_h$ | performance of pumped-hydro unit $h$. |
| $a_{bp}$ | commitment decision of thermal unit $t$ in subperiod $s$ of period $p$ [0/1]. |
| $b_{nsp}$ | consumption and generation by pumped-storage unit $b$ in load level $n$ of subperiod $s$ of period $p$. |
| $c_{nsp}$ | power consumption by pumped-hydro unit $h$ in load level $n$ of subperiod $s$ of period $p$. |
| $d_{nsp}$ | defect of power reserve in subperiod $s$ of period $p$. |
| $g_{hbp}$ | generation of hydro unit $h$ in load level $n$ of subperiod $s$ of period $p$. |
| $i_p$ | unavailability by scheduled maintenance for thermal unit $t$ during period $p$ [0/1]. |
| $n_{nsp}$ | non served and interruptible power in load level $n$ of subperiod $s$ of period $p$. |
| $P_{cp}$ | consumption of thermal plant $c$ in period $p$. |
| $P_{nsp}$ | generation of thermal unit $t$ in load level $n$ of subperiod $s$ of period $p$. |
| $R_{hp}$ | hydro energy reserve of hydro unit $h$ at the beginning of period $p$. |
| $S_{cp}$ | fuel storage capacity of thermal plant $c$ at the beginning of period $p$. |
| $e_{asp}$ | auxiliary variable in the equation of contiguity among maintenance periods. |
| $m_{rp}$ | reserve margin in period $p$. |
slack variables for maintenance constraints.

B. Formulation

1) Objective function of the FIRST stage

This objective function represents the sum of fuel costs (including independent and linear terms of the heat rate and O&M variable costs) plus startup costs plus storage costs of fuel stocks plus some penalties for non served power, interruptibility, and reserve margin defect, for all the load levels, subperiods and periods of the time scope.

\[
\sum_{p=1}^{P} \sum_{t=1}^{T} \sum_{p=1}^{P} \sum_{n=1}^{N} D_{np} \sum_{m=1}^{M} \left[ v_{p} P_{np} + v_{p} P_{np} + u_{p} P_{np} \right] + \sum_{p=1}^{P} \sum_{t=2}^{T} \sum_{p=1}^{P} \left[ n(a_{p} - a_{p}) + \sum_{c=1}^{C} \sum_{p=1}^{P} \sum_{s=1}^{S} \sum_{n=1}^{N} D_{np} \sum_{d=1}^{D} \left( w_{dnp} + w_{dnp} \right) \right] + \sum_{p=1}^{P} \sum_{t=1}^{T} \sum_{p=1}^{P} \left[ w_{np} + w_{np} \right] + W \sum_{p=1}^{P} \sum_{t=1}^{T} \sum_{p=1}^{P} f_{np}
\]

2) Objective function of the SECOND stage

This function is the sum of the slack variables of the constraint (4), that is to say, the sum of the differences between reserve margins of consecutive periods.

\[\sum_{p=1}^{P} h_{p} + a_{p}\]

3) Maintenance scheduling constraints

• Relation between consecutive periods reserve margins

The thermal reserve margin of each period is the total available thermal capacity divided by the period peak load.

\[m_{r}-m_{r+1}+h_{p}-a_{p}=0\]

• Duration of MS

The units will be turned off for an integer number of periods by maintenance.

\[\sum_{p=1}^{P} i_{p} = M_{t}\]

• Maximum number of units in simultaneous maintenance for the same plant

\[\sum_{t \in c} i_{p} \leq g\]

• Maximum system outage capacity simultaneously in maintenance

\[\sum_{t \in c} i_{p} \leq \frac{\sum_{t \in c} i_{p} P_{t}}{P_{t}}\]

4) Contiguity constraints

The MS problem requires contiguity to be modeled when the duration of maintenance works exceeds the time period of the models. Only contiguous solution of periods in maintenance for every unit can be feasible. In this problem contiguity is modeled using auxiliary constraints and continuous variables. The constraints are formulated in the following way and complemented by the convexity and bounds of continuous variables.

\[i_{p} \leq \sum_{p \in M_{t}} \left( \xi_{ip} + \xi_{p} \cdot 1 + \ldots + \xi_{ip} + M_{t} \cdot 1^L \right)
\]

\[\sum_{p \in M_{t}} \xi_{ip} = 1\]

The variable \(\xi_{ip}\) is a continuous variable always zero except for one of the periods of the model scope. This fact is used by the constraint (8) to establish the logical implication between these two variables.

Other typical maintenance constraints as manpower and material resources limits or sequence constraints ("precedence", "frozen time", etc.) could be easily included within this formulation.

5) Fuel scheduling constraints

The stock level at the beginning of each period is a function of the consumption and purchases during the period and the stock of the previous period, for each thermal plant.

\[S_{cp} + C_{cp} \cdot P_{cp} \leq S_{cp+1}\]

\[P_{cp} = \sum_{p=1}^{P} \sum_{s=1}^{S} \sum_{n=1}^{N} \sum_{t=1}^{T} D_{np} \sum_{k=1}^{K} \left( a_{k} P_{np} + a_{k} P_{np} + a_{k} P_{np} \right)\]

6) Hydro scheduling constraints

For each hydro unit, the hydro reserve level at the beginning of each period is a function of the previous level, the hydro inflow and pumping and generation on that period. The initial and final hydro reserves are specified by the user.

\[R_{hp} + A_{hp} + h_{p} \sum_{s=1}^{S} \sum_{n=1}^{N} D_{np} \sum_{t=1}^{T} \left( h_{np} + h_{np} \right) \geq R_{hp+1}\]

6) Operation constraints

• Reserve margin

A power reserve margin for the peak load level of each subperiod must be met. This constraint represents the condition imposed to provide some amount of power available to account for increments in demand or failures of committed generation units.
The model establishes the balance between generation and demand at any load level including non served power and interruptibility.

\[ \sum_{t \in d} p_{tsp} + \sum_{h \in d} h_{hp} + \sum_{b \in d} t_{bwp} \geq d_{twp} (1 + R) \] (13)

### Generation-demand balance

- **Pumped-storage units**
  Balance between pumped and generated energy by pumped-storage units in a period and a reservoir limit imposed to the pumped energy.

\[ h_{bwp} \sum_{s=1}^{S} \sum_{n=1}^{N} D_{nwp} b_{nsp} = \sum_{s=1}^{S} \sum_{n=1}^{N} D_{nwp} b_{nsp} \] (15)

\[ \sum_{s=1}^{S} \sum_{n=1}^{N} D_{nwp} b_{nsp} \leq V_b \] (16)

### Generation constraints

- **For each thermal unit the maximum generation is less than the maximum available capacity and the minimum generation is greater than the minimum load. Thermal unit commitment related constraints state that the unit’s output during higher load levels must be larger than its generation in lower load levels and that the commitment decision in a higher load subperiod (weekdays) must be greater than those in a lower load subperiod (weekends).**

\[ p_{tsp} \leq p_{nsp} (1 - q_t) \] (17)

\[ p_{t1wp} \leq p_{twp} (1 - q_t) \] (18)

\[ p_{t1wp} \leq p_{nwp} \] (19)

\[ a_{twp} \leq a_{nwp} \] (20)

\[ a_{t1wp} \leq a_{n1wp} \] (21)

### Bounds of variables

\[ i_{twp} = \{ 0, 1 \} \]

\[ 0 \leq e_{nwp} \leq 1. \]

\[ S_c \leq S_{cwp} \leq \bar{S}_c \]

\[ h_{bwp} \leq h_{nwp} \leq \bar{h}_{bwp} \]

\[ e_{nwp} \leq e_{nwp} \leq \bar{e}_{nwp} \]

\[ R_{hwp} \leq R_{hwp} \leq \bar{R}_h \]

\[ a_{twp} = \{ 0, 1 \} \]

**V. IMPLEMENTATION**

The model has been implemented in a powerful language for easy formulation of optimization problems called GAMS [1]. This high level language allows a fast and compact modeling (1500 code lines for this model) with no detriment of the writing and optimization time, to easily call the optimizers, in this case OSL [6], capable of solving large scale mixed integer problems.

For the Spanish electric system shown in this paper the optimization problem has more than 46000 constraints, 37300 variables (3692 of them binary) and 171800 non zero elements. The relaxed linear optimization is carried out through an interior point method called primal-dual predictor-corrector, changing to simplex once optimum for a basic solution has been reached. Crossover option is used for restarting from optimal continuous solution. The integer search is done with the b&b method, bounded by the relative optimality criterion (OPTCR), representing the relation (%) between the best integer solution found so far and the best possible integer solution. In this case, OPTCR is set to 0.003 for the first stage, setting it to 0.5 to reach an optimal solution after the second stage. Available in OSL, strategies 1 and 2 are selected. They perform probing only on satisfied 0-1 variables and using previous integer solution founded.

Preprocessing and probing techniques can be very effective in improving the performance of a mixed integer optimization [13]. In effect, the idea is reformulate the problem so as to obtain reductions in optimization time (linear and mixed integer), and in the gap between the linear and the integer solution.

In this case, setting as variable the unavailability of generation units instead of availability, produces advantages in both. The implementation is obvious. The linear optimization time is reduced about 30%, and the number of nodes evaluated to find the integer solution has been also decreased.

Scalation is another important feature with effects on large dimension optimization. That means, that variables, its derivatives and constraints should be near 1. For the system representation, kTcal, GW, TWh and Gptas (1S = 125 ptas) are used.

**VI. CASE STUDY**

The Spanish electric power system has about 71 thermal generators (8 nuclear with 7401 MW, 36 coal with 10675 MW and the remaining oil/gas with 7910 MW; totally 25986 MW) grouped in 16 thermal plants. Their production is about 80% of the total generation.
There are 130 hydro units (70 with capacity greater than 5 MW). For this study, all the hydro units are grouped into one, with capacity the sum of the all of them and considering only one reservoir. They produce as an average about 20% of the total generation, ranging between 13% and 28%, depending on the hydrology. There are 8 pumped storage units, but their impact is minimum (1%).

The analysis is done for 1994, when the maximum peak load was 25551 MW and the energy demand for the whole year was 145670 GWh. The system demand reaches the peak in the 9th week, and the minimum in the 41st week with 15155 MW. The profile shows is high in fall and winter and decrease in spring and summer.

The annual MS obtained for 1994 is presented in Fig. 3. TOC1 is 328162.8 millions of ptas ($2625 millions) and TOC2 is 338007.7 millions of ptas ($2704 millions), when \( p \) is 3570. More reliable solutions can be obtained increasing \( p \). In this case and due to the dimension of the problem, \( p \) can not be reduced.

Optimization times are very different. The first stage needs 2500 s to reach the optimal point and the second stage takes 7 times more. Time is expressed in seconds for a workstation Axil 311 Model 5.1 with Derformance of 83 SPECfp92 and 65 SPECint92.

VII. CONCLUSIONS

It has been reported a successful algorithm for solving large optimization MS problems using a goal programming method. Two important criteria have been optimized: system operating costs and leveling thermal reserve margins. The system representation is also relevant because of detailed modeling of maintenance and fuel scheduling decisions, thermal units operation and unit commitment. It is also new because of its application to a real power system not a study case.

The advantages of a multicriteria approach have been demonstrated by comparing the effects that costs and reliability have on each other in thermal plants maintenance scheduling. It has been shown that different optimization criteria give different optimal MS solutions, the more antagonistic the more different. This methodology allows the system planner to choose explicitly the compromise between both criteria using a control parameter determining the increase in costs allowed to levels the reliability of the system.

The easy formulation of this goal programming sequential method using GAMS is also relevant and new, as well as the easy way to call the optimizer and its optimization options.

Further work relating uncertainty in thermal units forced outages and system load can be done to show that the MS solution obtained by this method has a good behavior under probabilistic reliability measures such as LOLP or LOLE.

VIII. ACKNOWLEDGMENTS

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IX. REFERENCES


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Andrés Ramos was born in Guadramiro, Spain, in 1959. He received the degree of Electrical Engineer, from the Universidad Pontificia Comillas, Madrid, Spain, in 1982 and the Ph.D. degree of Electrical Engineer, from the Universidad Politécnica de Madrid, Madrid, Spain, in 1990. From 1982 to 1984 he was a junior research staff at the Instituto Tecnológico para Postgraduados. From 1984 up to now he is a senior research staff at the Instituto de Investigación Tecnológica from the Universidad Pontificia Comillas. Visiting scholar at Stanford University in 1991-1992 academic year.

His areas of interest include operations, planning and economy of electric power systems, operations research applied to power systems and software development.
Discussion

K.P. Dahal, C.J. Aldridge, J.R. McDonald (Centre for Electrical Power Engineering, University of Strathclyde, Glasgow, UK): The authors are to be congratulated for an interesting paper on generator maintenance scheduling for a real-sized system with multiple objectives using goal programming. We would like to raise the following points arising from the approach and results presented in the paper.

1. The paper uses two sequential optimisation stages: I. minimising system operating cost, II. levelling reserve margin by introducing an upper bound of the system operating cost. It has been reported in the literature that minimising production cost (which is the main part of the operating cost for thermal plants) is an insensitive objective for generator maintenance scheduling. For example Yameyee et al. [15] report the findings of Zürn and Quintana that for a practical system with different maintenance plans the production costs of the most expensive and least expensive schedules differ by only 0.08%. Have the authors experienced this observation during the problem solution in the first stage? What are the merits of reversing the order of the optimisation criteria by levelling the reserve in the first stage and minimising the system cost subject to an upper bound on the reserve differences in the second stage?

2. In the reliability criterion optimisation in the second stage, the constraint on the total operating cost is taken as $\text{TOC2} \leq \text{TOC1}(1+\beta/100)$, with $\beta$ determining the trade-off between operating cost and reliability. Could the authors clarify the reasons why $\beta$ cannot be reduced below 3% for the case study? Furthermore, can the increase in reliability calculated for $\beta=3\%$ and above be quantified?

The authors comments on these questions would be gratefully appreciated.

Closure was not provided by the Author.